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THE TRANSITION CURVE

OR

CURVE OF ADJUSTMENT

AS APPLIED TO THE ALIGNMENT OF RAILROADS

BY THE METHOD OF RECTANGULAR CO-ORDINATES AND BY DEFLECTION ANGLES

(OR POLAR CO-ORDINATES)

BY

N. B. KELLOGG

M. AM. SOC. C. E.

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PREFACE

WHILE the theory of the transition curve as here developed is based upon the methods given by M. Nordling (see "Annales des Ponts et Chaussées," 1867), the equations for both the cubic parabola and the spiral appear in Professor Rankine's "Civil Engineering" (Ed. 1863), and are accredited to Dr. William Froude, undoubtedly ante-dating those of Nordling.

Translations of a portion of Nordling's demonstration have appeared from time to time, but only so far as related to connecting a straight line with a circular curve. That portion relating to connecting circular curves of different radii, by means of the cubic parabola, has not appeared in the form given by Nordling, so far as I am aware. The formulæ deduced in the latter case are of general application and equally true for connecting curve with curve, or curve with tangent, when proper values are introduced into the equations.

That of joining a tangent with a circular curve, by means of the transition curve, is a special case where one of the radii becomes infinitely great. Some of the recent spirals, adopted as curves of adjustment in railroad location, easily develop from the equations of the cubic parabola by making the proper substitutions in them.

Following the supposition indicated by M. Nordling, i.e., regarding $x = L$ and substituting L for x in the Nordling equations, the formulæ (42–46) were obtained by the writer during the summer of 1884, as were also § (2–9) inclusive, by an independent process, resulting in the verification of the formulæ invented by Froude in 1841 and published in 1861. Hence, it is thought that

Railroad Spiral" (an approximation
though giving good results as a
defective in not being expressed
g a law of uniform change of curv
points throughout its entire lengt
compound circular curve was pr
ly with that of the true spiral a
a, but was abandoned for the re
formulæ were found to be not eas
ified for arbitrary values of the
the computation of fractional chord
engineers, at the present time,
nt railway line without the use
f adjustment.

w simple methods of the calculu
the formulæ, but a knowledge of onl
es of algebra and trigonometry i

a *compound circular transition curve* and a
e offset, the origin of the compound circ
ne-half chord length *in advance* of that
terminus of the compound circular cur

their application. It is seldom necessary to use more than two terms in the formulæ involving series.

The *larger type* may be read independently of the smaller type. The latter may be used when the curve is extended beyond ordinary limits and greater accuracy in results is sought.

For field practice, formulæ (42–52), (60–61), are the ones with which, taken with the tables, one should become familiar; and these formulæ may be regarded as a summary of the essential part of the book.*

The "run off" is understood to be equal in length to that of the transition curve and coincident with it.

The transition or adjustment curve can rarely be applied to switches or turnouts to advantage, except where high speeds are maintained.

The more important formulæ are in **full-faced type.**

I am indebted to Professor H. I. Randall, C.E., of the University of California, and also to Mr. D. E. Hughes, C.E., for valuable suggestions. The latter is the author of an excellent paper on this subject.

N. B. K.

SAN FRANCISCO, January, 1904.

* By making all terms containing the factors m or f, *representing spiral offsets* in Probs. (I–V) *equal zero*, the spiral disappears from the formula and it stands for circular curves simple or compound as the case may be.

Except in a few cases, Greek letters have been used to designate angles, small Roman italics for lines and Roman capitals for points.

S. F. 1907.

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THE TRANSITION CURVE

§ 1. The true transition curve is one of which the radius of curvature, at its origin, is infinitely great; and at any other of its points, the radius of curvature is inversely proportional to the distance of the point, measured on the curve, from the origin; the product of the radius and distance being a constant for all points of the curve.

The object of introducing the transition curve between circular curves of different radii, or between a tangent and circular curve, as applied to the alignment of railroads, is to give centrifugal force an appreciable time to develop, from that due to one given radius of curvature to that of another, in a moving body passing from a circular path of one rate of curvature to that of another rate of curvature; and to develop simultaneously a force equal and opposed to the centrifugal force, neutralizing it at every point of the curve. If one of the given radii is made infinitely great, its curve becomes a tangent; and centrifugal force develops gradually from zero to that due to the other given radius of curvature, in the time it takes to traverse the transition curve; thus avoiding instantaneous development or "shock."

The opposing force is the horizontal component of a force due to gravity developed by inclining the vertical axis, passing through the center of gravity of the moving body, from a normal to the plane of rotation and towards the *center of rotation*.

From mechanics the expression for centrifugal force

The opposing horizontal force d
ig. 1):

$$w_{\prime} = \frac{we}{g},$$

which w is the weight of the movin
the path or gauge of the track, a
the path, or "cant," towards the
a distance g.

Since by the hypothesis the opp
h other in intensity,

$$f = w_{\prime}$$

$$\frac{we}{g} = \frac{wv^2}{32.2r_{\prime}} \text{ solvin}$$

$$e = \frac{gv^2}{32.2r_{\prime}} \text{ nearly}$$

---

* More correctly (Fig. 1) $e = \frac{hv^2}{\sqrt{(32.2)^2 r^2}}$.

in which h =
The result

If i denote the distance, measured on the curve, required to change the inclination or "cant" of the path one unit, $\frac{1}{i}$ will denote the change of cant in one unit of distance. Therefore, in a distance $OB_{,} = L_{,}$ the cant, or superelevation, will be:

$$\frac{L_{,}}{i} = e = \frac{gv^2}{32.2r_{,}} \tag{1}$$

from which

$$L_{,}r_{,} = \frac{gv^2 i}{32.2} \tag{2}$$

The second member of Eq. (2) is a constant, since all its factors are assumed constant, and represented by P, whence, **the fundamental equation of the curve:**

$$L_{,}r_{,} = P, \tag{3}*$$

which is the equation of a spiral, in which $OB_{,} = L_{,}$, Fig. 2, represents the required length of the transition curve, $DB_{,} = r$, the radius of the curvature common to the transition curve and the given circular curve where these curves become tangent each other. (See Fig. 2.)

GENERAL FORMULA.

§ 2. If in the figure we let $OB_{,}$ be denoted by $L_{,}$ and $OB_{,,}$ by $L_{,,}$, then

$$OB_{,} = L_{,} = \frac{P}{r_{,}};\ OB_{,,} = L_{,,} = \frac{P}{r_{,,}};$$

subtracting, letting $L_{,,} - L_{,} = L$, we have the *General Formula* for compound curves.

---

* Froude's Eq. (3), is in the form of $L = ei$, which is called by Rankine "The Curve of Adjustment." Any particular spiral is designated by the numerical value assigned to its constant P.

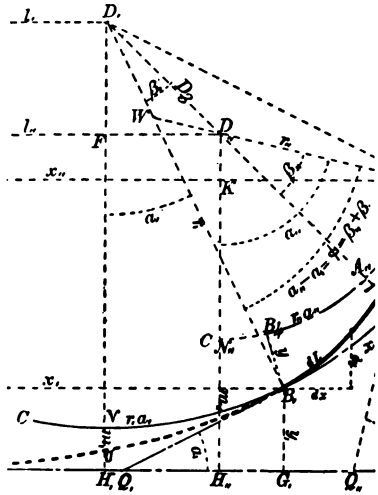


Fig. 2.

every condition of theory and offers

§ 3. In general let $L = \frac{P}{r} = Pr^{-1}$; differentiating $dL = Pr^{-2}dr$. We have from the calculus: $dL_{,} = rda$, whence

$$rda = Pr^{-2}dr,\ da = \frac{Pr^{-2}dr.}{r} = Pr^{-3}dr;\ \text{or}$$

$$a = \int Pr^{-3}dr = \frac{Pr^{-2}}{2} = \frac{P}{2r^2};$$

hence for any two arcs $a_{,,}$ and $a_{,}$ at radius $= 1$, we have:

$$a_{,,} = \frac{P}{2r_{,,}^2},\ a_{,} = \frac{P}{2r_{,}^2}; \tag{6}$$

$$a_{,,} - a_{,} = \frac{P}{2r_{,,}^2} - \frac{P}{2r_{,}^2} = \frac{P}{2}\left(\frac{1}{r_{,,}^2} - \frac{1}{r_{,}^2}\right),$$

or

$$a_{,,} - a = \frac{P}{2}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)\left(\frac{1}{r_{,,}} + \frac{1}{r_{,}}\right).$$

$$\frac{L}{2} = \frac{P}{2}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right); \tag{7}$$

substituting in value for $a_{,,} - a_{,}$, we have for the **Central Angle for Compound Curves expressed in arc at radius $= 1$.**

$$* \ a_{,,} - a_{,} = \frac{L}{2}\left(\frac{1}{r_{,,}} \pm \frac{1}{r_{,}}\right) = \phi = \beta_{,,} \pm \beta_{,} \tag{8}$$

---

* The values of $\beta_{,}$ $\beta_{,,}$ $a_{,}$ $a_{,,}$ and ϕ are expressed in arc at radius $= 1$. The degrees in an arc of a circle which is equal to the radius in length $= 57.295$. The arc of 1 degree at radius $= 1$ is $.01745+$. Hence the expression for any number of degrees in a given circular arc is: $A^{\circ} = \frac{\text{arc } A}{r\ 01745\ +}$.

e — sign when the *directions of curvatu*

EXAMPLE No. 1. Given $L = 150$; $r_{,,}$

) find arc $(\alpha_{,,} - \alpha_{,}) = \phi = \beta_{,,} + \beta_{,} =$

$$\text{arc } (\alpha_{,,} - \alpha_{,}) = \phi = \frac{150}{2}\left(\frac{1}{818.8} + \frac{}{2}\right.$$

$$= \text{arc of } 6^\circ\ 45'.$$

If in equation (8) we make $r_{,} = \infty$, ar

$$\alpha_{,,} = \frac{L_{,,}}{2r_{,,}}\ ;\ \alpha_{,,} =$$

ıe angle, expressed in arc, subtended

§ 4. DIFFERENCE IN LENGTH OF THE UM OF THE CIRCULAR ARCS SUBTENDING

The rectangular co-ordinates of the cente dius $r_{,}$, as $D_{,}$, referred to the origin O are: OH ıd of $D_{,,}$ $OH_{,,} = l_{,,}$, $H_{,,}D_{,,} = k_{,,}$; whence

$$\frac{l_{,,} - l_{,}}{k_{,} - k_{,,}} = \tan(\alpha_{,} + \beta_{,})\ ;$$

from either of these equations

$$\beta_{\prime\prime}{}^{\circ} = \alpha_{\prime\prime}{}^{\circ} - (\alpha_{\prime}{}^{\circ} + \beta_{\prime}{}^{\circ}), \tag{14}$$

with $\beta_{\prime\prime}{}^{\circ}$ reduced to arc, then

$$r_{\prime\prime}\beta_{\prime\prime} = A_{\prime\prime}B_{\prime\prime}$$

and

$$\beta_{\prime} = \alpha_{\prime\prime} - (\alpha_{\prime} + \beta_{\prime\prime}),\ r_{\prime}\beta_{\prime} = A_{\prime}B_{\prime}; \tag{15}$$

denoting the difference between L and $A_{\prime\prime}B_{\prime\prime} + A_{\prime}B_{\prime}$ by "d" we have

$$d = L - (A_{\prime\prime}B_{\prime\prime} + A_{\prime}B_{\prime}) = L - (r_{\prime\prime}\beta_{\prime\prime} + r_{\prime}\beta)$$

Placing

$$(r_{\prime\prime}\beta_{\prime\prime} + r_{\prime}\beta_{\prime}) = Lc,$$

we have

$$\text{"d"} = \text{L} - \text{Lc}. \tag{16}$$

§ 5. TRANSFORMATION TO RECTANGULAR CO-ORDINATES.

By the Calculus $\quad dy = dL \sin \phi$

$$dx = dL \cos \phi;$$

in which ϕ = any angle. By trigonometry,

$$\sin \phi = \frac{\phi}{1} - \frac{\phi^3}{1.2.3} + \frac{\phi^5}{1.2.3.4.5} - \text{etc.} \ldots \frac{\phi^n}{1.2.3..n};\ *$$

$$\cos \phi = 1 - \frac{\phi^2}{1.2} + \frac{\phi^4}{1.2.3.4} - \text{etc.} \ldots \frac{\phi_m}{1.2.3..m};$$

---

* This and the succeeding formula are for expressing the trigonometrical function of an arc in terms of the arc itself (see Chauvenet's *Trigonometry*, Chapter XIII, 1867).

m and n = *positive* whole numbers. m = even, n = odd.

$$\frac{dy}{dL} = \sin\phi = \sin(\beta_{\prime\prime} + \beta_{\prime}) = \sin B$$

$$\frac{L^2}{2P}\left(1 - \frac{L^4}{24P^2} + \frac{L^8}{1920P^4} - \text{etc.}\right.$$

$$\sin\phi = \frac{L}{2}\left(\frac{1}{r_{\prime\prime}} + \frac{1}{r_{\prime}}\right)\left[1 - \frac{L^2}{24}\left(\frac{1}{r_{\prime\prime}} + \frac{1}{r}\right.\right.$$

EXAMPLE No. 2. Given $L = 150$; $r_{\prime\prime} = 8$
ϕ.

$$\sin\phi = \sin(\beta + \beta) = \frac{L}{2}\left(\frac{1}{r_{\prime\prime}}\right.$$

$$\left[1 - \frac{L^2}{24}\left(\frac{1}{r_{\prime\prime}} + \frac{1}{r_{\prime}}\right)^2 + \text{etc.}\right]$$

$$= \sin(\beta_{\prime\prime} + \beta_{\prime}) = \frac{150}{2}\left(\frac{1}{818.8} + \frac{1}{2865}\right)\left[1 - \right.$$

$$\sin\phi = \sin(\beta_{\prime\prime} + \beta_{\prime}) = .11775 \times .997$$

$$dy = dL\left(\frac{L^2}{2P} - \frac{L^6}{48P^3} + \frac{L^{10}}{3840P^5} - \text{etc.} \ldots \frac{L^n}{NP^{\frac{n}{2}}}\right); \qquad (18)$$

integrating,

$$y = \frac{L^3}{6P}\left(1 - \frac{L^4}{56P^2} + \text{etc.} \ldots \frac{L_{,,}^{n-2}}{(n+1)\,NP^{\frac{n}{2}-1}}\right); \qquad (19)$$

or, since $P = \dfrac{L}{\dfrac{1}{r_{,,}} - \dfrac{1}{r_{,}}}$, with its origin at B, the ordinate of any point of the spiral, as

$$B_{,,}B_{,,,} = y = \frac{L^2}{6}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)\left[1 - \frac{L^2}{56}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)^2 + \text{etc.}\right]. \qquad (20)$$

EXAMPLE 3. Given $r_{,} = 2865$; $r_{,,} = 818.8$; $L = 150$, to find y by:

$$y = \frac{L^2}{6}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)\left[1 - \frac{L^2}{56}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)^2 + \text{etc.}\right] =$$

$$\frac{(150)^2}{6}(.00122 - .00035)\,[1 - \frac{(150)^2}{56}(.00122 - .00035)^2 + \text{etc.}]$$

$$y = \frac{22500 \times .00087}{6}\,[1 - 2500\,(.00087)_2 + \text{etc.} = 3.2625\,[.9997]$$

$$= 3.26225.$$

Or in terms of ϕ, since $L = 2r\phi$, and $\dfrac{1}{r} = \left(\dfrac{1}{r_{,,}} - \dfrac{1}{r_{,}}\right)$

$$y = \frac{2}{3}r_{,}\phi^2\left(1 - \frac{\phi^2}{14} + \right) = \frac{\dfrac{2\phi^2}{3}}{\left(\dfrac{1}{r_{,,}} - \dfrac{1}{r_{,}}\right)}\left[1 - \frac{\phi}{14} + \text{etc.}\right]. \qquad (20a)$$

$r_{\prime} = \infty$, $\frac{1}{r_{\prime}} = 0$, $L = L_{\prime\prime}$ the ordinate of any point

$$= \frac{L_{\prime\prime}^2}{6r_{\prime\prime}}\left(1 - \frac{L_{\prime\prime}^2}{56r_{\prime\prime}^2} + \text{etc.}\right) \quad \text{with origin at O} \qquad (21)$$

and $OG_{\prime\prime}$ as abscissa.

$$\cos\phi = \left(1 - \frac{L^4}{8P^2} + \frac{L^8}{384P^4} - \text{etc.}\right) \qquad (22a)$$

$$- \frac{L^4}{8P^2} + \frac{L^8}{384P^4} - \frac{L^{12}}{46080P^6} + \text{etc.} \ldots \frac{L^m}{MP^{\frac{m}{2}}}\Bigg)$$

$$- \frac{L^4}{40P^2} + \frac{L^8}{3456P^4} - \frac{L^{12}}{599040P^6} + \text{etc.} \ldots$$

$$\frac{L^m}{(m+1)\,MP^{\frac{m}{2}}}\Bigg); \qquad (23)$$

Example 4.

$$x = L\left[1 - \frac{L^2}{40}\left(\frac{1}{r_{\prime\prime}} - \frac{1}{r_{\prime}}\right)^2 \pm \text{etc. (other terms)}\right]$$

$$= 150\left(1 - \frac{22500 \times .0000007569 +}{40} \pm\right);$$

$$x = 150\ (.9996 \pm) = 149.94.$$

With origin at O, any abscissa, as

$$OG_{\prime\prime} = x_{\prime\prime} = L_{\prime\prime}\left(1 - \frac{L_{\prime\prime}^2}{40\,r_{\prime\prime}^2} + \frac{L_{\prime\prime}^4}{3456\,r_{\prime\prime}^4} - \frac{L_{\prime\prime}^6}{599040\,r_{\prime\prime}^6} + \text{etc.}\right); \quad (26)$$

also when $\phi = \alpha_{\prime\prime}$

$$x_{\prime\prime} = 2r_{\prime\prime}\alpha_{\prime\prime}\left(1 - \frac{\alpha_{\prime\prime}^2}{10} + \frac{\alpha_{\prime\prime}^4}{216} - \text{etc.}\right) \quad (26a)$$

§ 6. RECTANGULAR CO-ORDINATES AT THE OFFSET DISTANCE.

1°. **To find the abscissa and length of the curve.**

Let $y_{m\prime\prime}$ be the ordinate, $x_{m\prime\prime}$ the abscissa, and $L_{m\prime\prime}$ the length of the curve corresponding to $x_{m\prime\prime}y_{m\prime\prime}$; then a very nearly correct value of $y_{m\prime\prime}$ for central angles up to 50 degrees may be obtained as follows:

$OH_{\prime\prime} = x_{m\prime\prime} = x_{\prime\prime} - r_{\prime\prime}\sin\phi$: whence, with origin at O, (27)
by Eq. * page 7 and (27).

$$OH_{\prime\prime} = x_{m\prime\prime} = r_{\prime\prime}\phi\left(1 - \frac{\phi^2}{30} + \frac{\phi^4}{1080} - \ldots\right) \text{or} \quad (27a)$$

in terms of $r_{\prime\prime}$ and $L_{\prime\prime}$

$$OH_{\prime\prime} = x_{m\prime\prime} = \frac{L_{\prime\prime}}{2}\left(1 - \frac{L_{\prime\prime}^2}{120\,r_{\prime\prime}^2} + \frac{L_{\prime\prime}^4}{17280\,r_{\prime\prime}^4} \ldots\right) \quad (28)$$

and when the origin is at $B_{\prime}$

$$B_{\prime}A_{\prime} = x_f = \frac{L}{2}\left[1 - \frac{L^2}{120}\left(\frac{1}{r_{\prime\prime}} - \frac{1}{r_{\prime}}\right)^2 + \frac{L^4}{17280}\left(\frac{1}{r_{\prime\prime}} - \frac{1}{r_{\prime}}\right)^4 - \right] \quad (28a)$$

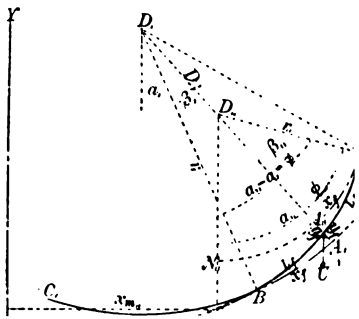
in which $L = L_{\prime\prime} - L_{\prime} = B_{\prime}B_{\prime\prime}$.

$$= L_f = \frac{x_f}{1 - \frac{L_m^2}{40}\left(\frac{1}{r_m} - \frac{1}{r_f}\right)^2} \text{ and } r_f = \frac{P}{L_f + L_f}$$

may write L_m for L_f and x_m for x_f, r_m fo
n (30) when the origin is at O; then

$$L_m = \frac{x_m}{1 - \frac{L_m^2}{40 r_m^2}}$$

To find the ordinate of a transition curve n
L_f (or f):



The method given in Equation (47) for finding the ordinate $y\ f$ is approximate and sufficiently close, within the limits of the tables. For a large central angle or great length of spiral a closer approximation is necessary.

Writing $y_{m''}$ for y, when origin is at O, we have by (19):

$$y_{m''} = \frac{L_{m''}^3}{6P}\left(1 - \frac{L_{m''}^4}{56P^2} + \text{etc.}\right) = \frac{L_{m''}^2}{6r_{m''}}\left(1 - \frac{L_{m''}^2}{56r_{m''}^2} + \text{etc.}\right); \quad (31)$$

with origin at B, then $B_fU = L_f = P\left(\frac{1}{r_f} - \frac{1}{r_{f'}}\right)$, and

$$A\,U = y_f = \frac{L_f^2}{6}\left(\frac{1}{r_f} - \frac{1}{r_{f'}}\right)\left[1 - \frac{L_f^2}{56}\left(\frac{1}{r_f} - \frac{1}{r_{f'}}\right)^2 + \text{etc.}\right] \quad (32)$$

$$\beta_{f'} = \frac{x_f}{r_{f'}}; \ \beta_{f''} = \phi - \beta_{f'} = (\alpha_{f''} - \alpha_{f'}) - \beta_{f'}.$$

This method of obtaining $\beta_{f''}$ and $\beta_{f'}$ may be used instead of Eq. 6 or 8.

§ 7. OFFSET DISTANCE.

In general, by Fig. (2), the offset distance.

$$f = y - R \text{ ver } \phi = y - R\,(1 - \cos\phi), \quad (33)$$

in which R depends on $\frac{1}{r_{f''}} - \frac{1}{r_{f'}}$ for its value, substituting for $\cos\phi$ its value $\left(\text{remembering that } R = \frac{P}{L}\right)$, and reducing, since

$$R\,(1 - \cos\phi) = R\left[1 - \left(1 - \frac{L^4}{8P^2} + \frac{L^8}{384P^4} - \text{etc.}\right)\right],$$

$$f = y - R \text{ ver } \phi = L\left[\left(\frac{L^2}{6P} - \frac{L^6}{336P^3} + \text{etc.}\right) - \right.$$

$$\left.\left(\frac{L^2}{8P} - \frac{L^6}{384P^3} + \text{etc.}\right)\right]; \quad (34)$$

$$- \frac{L^6}{2688P^3} + \text{etc.}\Big) = \frac{L}{P}\left(\frac{L^2}{24} - \frac{L^6}{2688P^2} + \text{etc.}\right); \quad (35)$$

since $\frac{L}{P} = \left(\frac{1}{r_{ii}} - \frac{1}{r_i}\right)$, we have for (36)

origin at B_i:

$$= \frac{L^2}{24}\left(\frac{1}{r_{ii}} - \frac{1}{r_i}\right)\left[1 - \frac{L^2}{112}\left(\frac{1}{r_{ii}} - \frac{1}{r_i}\right)^2 + \text{etc.}\right] \quad (37)$$

Given $r_{ii} = 818.8$; $r_i = 2865$; $L = 150$; to find first, when the circular curves r_{ii} and r_i turn in the by (125):

$$= \frac{L^2}{24}\left(\frac{1}{r_{ii}} - \frac{1}{r_i}\right)\left[1 - \frac{L^2}{112}\left(\frac{1}{r_{ii}} - \frac{1}{r_i}\right)^2\right] =$$

$$\times .00087\left[1 - \frac{(150)_2\,(.00087)^2}{112}\right] = .815.$$

O, when ϕ becomes α_{ii}, L becomes L_{ii} and f becomes

$$_{ii} = m_{ii} = \frac{L_{ii}^2}{24r_{ii}}\left(1 - \frac{L_{ii}^2}{112r_{ii}^2} + \text{etc.}\right) \quad (38)$$

se circular curves turn in opposite directions or are

EXAMPLE 6. Given $L = 150$; $r_{,} = 2865$; $r_{,,} = 818.8$; to find the distance between the centers of curves with radii $r_{,,}$ and $r_{,}$. First, if curves turn in the same direction (Fig. 2):

$$D_{,,}D_{,} = r_{,} - \left[r_{,,} + \frac{L^2}{24}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)(1 - \text{etc.})\right].$$

$$D_{,,}D_{,} = r_{,} - (r_{,,} + f) = 2865 - \left[818.8 + \frac{(150)^2}{24} + .00087\right];$$

$$D_{,,}D_{,} = 2865 - [818.8 + .815] = 2865 - 819.615 = 2045.385.$$

Second, if curves are reversed (or turn in a contrary direction) (Fig. 9), then:

$$D_{,,}D_{,,,} = \left[r_{,} + \left(r_{,,} + \frac{1}{24}\left(\left(\frac{L_{,,}^2}{r_{,,}} + \frac{L_{,}^2}{r_{,}}\right) - \frac{1}{112}\left(\frac{L_{,,}^4}{r_{,,}^3} + \frac{L^4}{r_{,}^3}\right)\right)\right)\right] \quad (41a)$$

$$\times \sec K_{,,}D_{,}D_{,,,} = [r_{,} + (r_{,,} + m_{,,} + m_{,})] \sec K_{,,}D_{,,}D_{,,,};$$

$$\tan K_{,,}D_{,}D_{,,,} = \frac{x_{m_{,,}} + x_{m_{,}}}{r_{,} + r_{,,} + m_{,,} + m_{,}}.^{*} \quad (41b)$$

PRACTICAL FORMULÆ.

§ 8. If, in the foregoing values of y, x, $\frac{dy}{dL}$, and f we omit terms in the bracket after the first or second, we have for central angles of 20°, or less, the following practical formulæ for uniting circular curves of different radii by means of a spiral arc:

$$\mathbf{B_{,,}B_{,,,}} = \mathbf{y} = \frac{\mathbf{L}^2}{6}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right); \quad (42)$$

See Eq. (32)

$$\mathbf{A_{,}U} = \mathbf{y}_f = \frac{\mathbf{L}_f^2}{6}\left(\frac{1}{r_f} - \frac{1}{r_{,}}\right)\left[1 - \frac{\mathbf{L}_f^2}{56}\left(\frac{1}{r_f} - \frac{1}{r_{,}}\right)^2 + \text{etc.}\right. \quad (42a)$$

$$\mathbf{B_{,}B_{,,,}} = \mathbf{x} = \mathbf{L}\left[1 - \frac{\mathbf{L}^2}{40}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)^2 + \right. \quad (43)$$

See Eq. (28a)

$$\mathbf{B_{,}A_{,}} = \mathbf{x}_f = \frac{\mathbf{L}}{2}\left[1 - \frac{\mathbf{L}^2}{120}\left(\frac{1}{r_{,,}} - \frac{1}{r_{,}}\right)^2 + \ldots\right] \quad (43a)$$

---

* When $O_{,}O_{,,} = 0$.

$$_{//} = L = x\left(1 + \frac{x^4}{40P^2}\right); \text{ nearly} \tag{44}$$

$$_{f} = \frac{L + 10x_f}{12} \text{ see Eq. (29)} \tag{44a}$$

$$B_{//} = \sin\phi = \sin(\beta_{//} + \beta_{/}) = \tag{44b}$$

$$\frac{1}{_{//}} + \frac{1}{r_{/}}\Big)\Big[1 - \frac{L^2}{24}\left(\frac{1}{r_{//}} + \frac{1}{r_{/}}\right)^2 + \text{etc.}$$

$$\text{at radius} = 1) = \frac{L}{2}\left(\frac{1}{r_{//}} + \frac{1}{r_{/}}\right) = \text{arc } \phi: \tag{44c}$$

$$\text{arc } \phi \times 57.3^\circ = \phi^\circ$$

$$A_{/}A_{//} = f = \frac{L^2}{24}\left(\frac{1}{r_{//}} - \frac{1}{r_{/}}\right), \tag{45}$$

latter equation we find:

$$\frac{_{/}B_{//}}{2} = \frac{L}{2} = \sqrt{6f \div \left(\frac{1}{r_{//}} - \frac{1}{r_{/}}\right)} \tag{46}$$

For uniting a tangent with a circular curve by the use of the spiral,

we have $\frac{1}{r_{,}} = o$, in (42–47);

whence: by Fig. 2, for any distance on the spiral, as $L_{,,,}$ with radius $r_{,,,}$ the corresponding

$$\text{Ordinate } G_{,,}B_{,,} = y_{,,} = \frac{L_{,,}^2}{6r_{,,}} \qquad (48)$$

$$\text{and the abscissa } OG_{,,} = x_{,,} = L_{,,}\left[1 - \frac{L_{,,}^2}{40r_{,,}^2} + \right] \qquad (49)$$

The sine of central angle of the transition curve equals

$$\sin N_{,,}D_{,,}B_{,,} = \sin \alpha_{,,} = \frac{L_{,,}}{2r_{,,}}\left(1 - \frac{L_{,,}^2}{24r_{,,}^2}\right) \qquad (50)$$

$$\text{arc } N_{,,}D_{,,}B_{,,} = \alpha_{,,} = \frac{L_{,,}}{2r_{,,}}, \text{ arc } \alpha_{,,} \times 57.3^\circ = \alpha_{,,}^\circ$$

$$\text{the offset } N_{,,}H_{,,} = m_{,,} = \frac{L_{,,}^2}{24r_{,,}} \qquad (51)$$

$$\tfrac{1}{2} \text{ the length of spiral} = \frac{L_{,,}}{2} = \sqrt{6m_{,,}r_{,,}} \qquad (52)$$

$$y_{m_{,,}} = \frac{L_{m_{,,}}^3}{6P}\left(1 - \frac{L_{m_{,,}}^4}{56P^2} + \text{etc.}\right) = \frac{L_{m_{,,}}^2}{6r_{m_{,,}}}\left(1 - \frac{L_{m_{,,}}^2}{56r_{m_{,,}}^2} + \text{etc.}\right); \qquad (52a)$$

$$OH_{,,} = x_{m_{,,}} = x_{,,} - r_{,,} \sin \alpha_{,,,} \text{ or} \qquad (a)$$

$$OH_{,,} = x_{m_{,,}} = \frac{L_{,,}}{2}\left(1 - \frac{L_{,,}^2}{120r_{,,}^2} + \frac{L_{,,}^4}{17280r_{,,}^4} \cdots\right) \qquad (52b)$$

$$O_{,}d \text{ or } O_{,}I = t = x_m + (r_{,,} + m) \tan \tfrac{1}{2} I \text{ (see Fig. 5)} \qquad (b)$$

If $D_{,,}$ and $D_{,}$ denote the "degrees" of the curves (determined by 100 feet of their length) corresponding to the radii, $r_{,,}$ and $r_{,}$; then, since $L_{,,}$ and $D_{,,}$ and $L_{,}$ and $D_{,}$ are inverse functions of $r_{,,}$ and $r_{,}$, we have

$$L_{,,} - L_{,} = L = L_{,,}\frac{D_{,,} - D_{,}}{D_{,,}} \text{ and } \frac{1}{r_{,,}} - \frac{1}{r_{,}} = \frac{L_{,,}}{P}\left(\frac{D_{,,} - D_{,}}{D_{,,}}\right)$$

the *difference* between curvature at l
same as when $\beta_{,} = 0$, and $\beta_{,,} = \frac{L}{2}$
y can be computed and laid off from
axis of abscissa with $B_{,}$ as origin, the
If we now conceive this new axi
o a radius $r_{,}$ the curvature of the tr
point will be increased by the san
rdinates may, without serious error,
he arc $B_{,}A_{,}$ and establish points of
The same reasoning will apply if
becomes tangent and values of x an
t with $B_{,,}$ as origin, except that the
urve would be convex to $B_{,,}A_{,,}$.
however, be equal to those of the co
rom $B_{,}$. If $r_{,,}$ now resume its orig
ature of the transition curve at a
hat of the circular curve with radiu
ature it had in a contrary direc
nfinitely great and $B_{,,}A_{,,}$ a straight
ng $\beta_{,,}$ and $r = \frac{P}{\ }$. data may be take

for x and y, their values may be laid out simultaneously with corresponding equal chord measurements along the transition curve.

The principle enunciated in the paragraph preceding, enables us to prepare a table of deflection angles according to the following method:

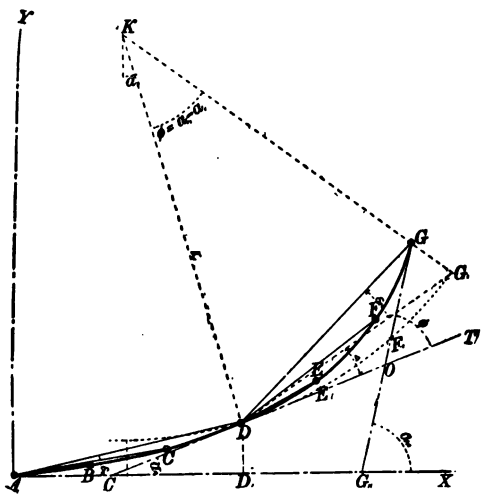


Fig. 3.

Referring to Fig. 3. If the deflection angles from AX to any point be denoted by $\delta\delta_{,}\delta_{,,}$, it will be found by computation that any angle as

$$\mathrm{DAX} = \delta_{,} = \frac{a_{,}}{3} = \frac{\mathrm{L}}{6\mathrm{r}_{,,}} \text{ (nearly)},^{*} \tag{53}$$

in which $r_{,,} = DK$ and $L = AD$.

$$\mathrm{ADC}_{,} = \mathrm{DCX} - \mathrm{DAX} = a_{,} - \frac{a_{,}}{3} = \frac{\mathrm{L}}{2\mathrm{r}_{,,}} - \frac{\mathrm{L}}{6\mathrm{r}_{,,}} = \frac{\mathrm{L}}{3\mathrm{r}_{,,}} = \tag{54}$$

---

* Since $\frac{DD_{,}}{AD_{,}} = \frac{y_{,,}}{x_{,,}} = \tan\frac{a_{,}}{3}$ (nearly).

rom D' to the point E, for the circular curv
he same way with D as origin.

$$GDT = \delta_{//} + \Delta_{//}; = \omega.$$

he angle

$$DGG_{//} = 2\,\delta_{//} + \Delta_{//} = (\alpha_{//} - \alpha_{/}) - (\delta_{//}$$

f we add $(\delta_{//} + \Delta_{//})$ to both members of the e

$$GOT = \alpha_{//} - \alpha_{/} = \phi = 3\,\delta_{//} +$$

n which $\Delta_{//} = L\Delta$, Δ = the deflection for a u
he circular curve with radius $r_{//}$, and L the
oint laid off on DG.

FOR CONVENIENCE IN FIEL

Eq. (53) may be reduced to degrees
ollowing form:

$$\delta^\circ = \frac{L}{6r}\,57.3^\circ, \text{ whence t}$$

$$* \quad \omega^\circ = D\Delta L \pm \frac{L}{6r}\,57.3^\circ \text{ in}$$

he instrument point is the origin of L.
Δ = deflection angle per foot from
rcular curve.
D = degree, or rate of curvature at

This equation can be still further simplified in application by the following reduction in $\delta^\circ = \frac{L}{6r} 57.3^\circ$, let r = radius due to L measured from the position of the instrument to the point of spiral to be located.

N = the number of chords of equal length (each subtending one degree change in rate of curvature) in the length of L of the spiral. Then if r_0 = the radius of a one-degree curve,

$$r = \frac{r_0}{N} \text{ and } \delta^\circ = \frac{LN\ 9.55^\circ}{5730} = LN \times 0.00166^\circ \quad (60)$$

or

$$\delta' = LN\ 0^\circ 00.1'.$$

If we wish the change in rate of curvature to be any other than one degree to the chord, and denote this change by C, C = the change in degree of curvature, due to one-chord length, and may be either a fraction or whole number; then

$$\delta' = LNC \times 0^\circ 00.1'; \quad (60a)$$

$$\text{or since } C = \frac{D_l}{N},\ \delta' = LD_l \times 0^\circ 00.1';^* \quad (60b)$$

and if one of the chords be fractional, and we indicate the fractional part by $\frac{n}{F}$ in which F is the number of parts into which a chord is divided, and n the number of such parts taken,

$$\delta' = L\left(N + \frac{n}{F}\right) C \times 0^\circ 00.1', \text{ hence} \quad (60c)$$

$$\omega' = \mathbf{L}\left[\mathbf{D}\Delta \pm \left(\mathbf{N} + \frac{\mathbf{n}}{\mathbf{F}}\right) \mathbf{C} \times 0^\circ 00.1'\right] \quad (61)$$

is a general formula for the deflection from a tangent, at any point of the transition curve, to locate any other point of the transition curve.

---

* See Appendix.

$$\omega' = L\left[D\Delta + (D_f - D)\ 0°00.1'\right.$$

With instrument at A, $D\Delta = 0$.

place the line of sight on tangent at any poi
backsight on last instrument point, formula
tangent becomes

$$\omega' = L\left[D\Delta + 2\left(N + \frac{n}{F}\right) C\,0°00.1'\right.$$

unning towards G (Fig. 3), D being the rat
it instrument point, and L the length of spi

. (57) $\omega' = L\left[D\Delta - 2\left(N + \frac{n}{F}\right) C\,0°00.1'\right]$

pplying Eq. (61), the more frequent the ch
nearly will the resulting curve agree with
; their distance apart to be not more tha
hange points" include a central angle of mor
earer value for the second term in the brack

$$\left(N + \frac{n}{F}\right) C\,0°\,00'.0998.$$

MPLE 7. Given, $D = 2$, $\Delta = .3'$, $L = 125$,
Chords = 50 ft., to find the deflection from
te a point $E + 25$ (see Fig. 4). By formulæ

$$\omega' = L\left[D\Delta + \left(N + \frac{n}{F}\right) \times 1 \times 0°00.1'\right.$$

uting values given above,

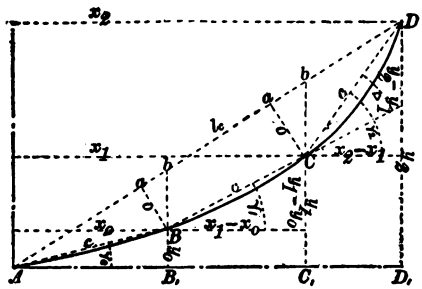


Fig. 4.

"n" having any value, whole number or fractional.

With A as origin we obtain, by (62) the deflection angles to the points B, C, D, E, etc., to any change point, as C, where the degree, or rate of curvature, is D. From C, with backsight on A, deflect $\alpha - \delta_n$ (in which $n = 2$, and α = the central angle of the spiral from A to C) to get on tangent at C, then to locate any point, as E (not shown on Fig. 4), from a tangent at C, deflect

$$* \ \omega = LD\Delta + \delta_n \tag{63}$$

δ_n being the deflection angle due to n chords from C, corresponding to the same number of chords from A.

To get on tangent at E:

With backsight on C, deflect $LD\Delta + (\alpha - \delta_n)$, in which $n = 2$, $L = CE$, and α corresponds to the central angle of the spiral from A *for a length* L.

The process is the same as with Eq. (61), though the second ~~factor~~ term of the second member is different.

---

* It will be seen that the differences between this method and that of Eq. (61), for a curve with $L = 400$ ft. and $\delta = 16°$ is 0°00′03″.

The difference increases with the central angle α. The above method, with change points 200 to 300 ft. apart, is quite accurate and the best for preparing a set of tables — though not so easily applied in field computations as (60) or (61). By assuming "change points" 100 to 250 ft. apart, *the deflection tables* of this book may be extended indefinitely.

EXAMPLE 8. Given ~
om A.

$$\tan \delta_e = \frac{4.65}{199.91} = .023260 \text{ or } \delta_e = 1$$

To get on tangent at C, at which point the tot
$00' - 1° 19.95' = 2° 40.05'$. Hence with the
cksight on A, deflect $2° 40.05'$ to get on tan
te of curvature is $D = 4° 00'$.

From tangent at C to locate some point E, w
00 ft. from C, then

$$\omega = LD\Delta + \delta_n = 200 \times 4 \times .3' + 1° 19.$$

nd to get on tangent at E, with backsight on (

$$\omega = LD\Delta + (\alpha - \delta_e) = 200 \times 4 \times .3' + 2°$$

ORDINATES.

§ 10. To determine the ordinates o, o_1,
as l_0, let $aB = o$, $B_1b = s$, $AB_1 = x_0$, $B_1B =$
Then from Fig. 4, $\frac{s}{x_0} = \tan \gamma$, $s = x_0 \tan \gamma$,

$\frac{o}{s - y_0} = \cos \gamma$, $o = (s - y_0) \cos \gamma$, or since s

$$o = (x_0 \tan \gamma - y_0 \cos \gamma = x_0 \sin \gamma$$

For the distances Aa, etc., $Aa = A_1B_1$
In the same way we may

A and D. Let the length of any chord be c_2, c_4, c_n, in which 2, 4, n indicate the 2d, 4th and nth increment.

$$c_2 = (x_2 - x_1) \sec \gamma_2; \quad c_4 = (x_4 - x_3) \sec \gamma_4;$$

$$c_n = (x_n - x_{n-1}) \sec \gamma_n. \tag{66}$$

EXAMPLE 9. Given $x_2 = 209.65$; $x_1 = 120$; $\gamma_2 = 4° 37'$, to find the length of chord, $c_2 = (x_2 - x_1) \sec \gamma_2 = 89.65 \times 1.0032 = 89.93$. For the length of long chord (from origin), $x_2 = 209.65$; $\gamma = 2° 27'$; $x_2 \sec \gamma = 209.65 \times 1.009 = 209.84$.

It is to be observed that the superelevation of the outer rail, in the use of the transition curve, may be made greater or less than that which has been assumed in computing the tables; the only effect it will have is to diminish or increase the assumed value of "i," which is equivalent to increasing or diminishing the velocity, since i and v are inverse functions of each other in the constant P, i.e., it makes the rate of rise of the outer rail to effect superelevation a little greater or less. It is, however, best to introduce the average velocity of the express or fast passenger trains in constructing the tables. Where the location is so constrained that the EC's and BC's of the circular curves are quite close together, it may be necessary to give "i" a smaller value than would be otherwise desirable. A value of 300 or 400 is sufficient for adjustment, and good results may be obtained with a value of 200 when the radius is not greater than 573 feet, since v usually is made to decrease with r.

The beginning and end of the transition curve should be marked by permanent points.

Degree of Curve	Radius	Reciprocal	Degree of Curve
D	r	$\frac{1}{r}$	D
0°30′	11460.	.00008726	8°00
1°00′	5730.	.00017453	9°00
1°30′	3820.	.00026179	10°00
2°00′	2865.	.00034906	11°00
2°30′	2292.	.00043633	12°00
3°00′	1910.	.00052359	13°00
3°30′	1637.1	.00061086	14°00
4°00′	1432.5	.00069808	15°00
4°30′	1273.3	.00078540	16°0(
5°00′	1146.	.00087267	17°0(
5°30′	1041.8	.00095993	18°0
[illegible]	955	.00104712	19°0

PROBLEM I.

TO FIND THE SEMI-TANGENTS.

§ 11. Given a circular curve whose radius $= r_{,,}$; the intersection angle $= I$; the semi-tangent $= T$, to unite it with the tangents by means of transition curves whose lengths are $L_{,}$ and L and offsets are $m_{,}$ and m respectively.

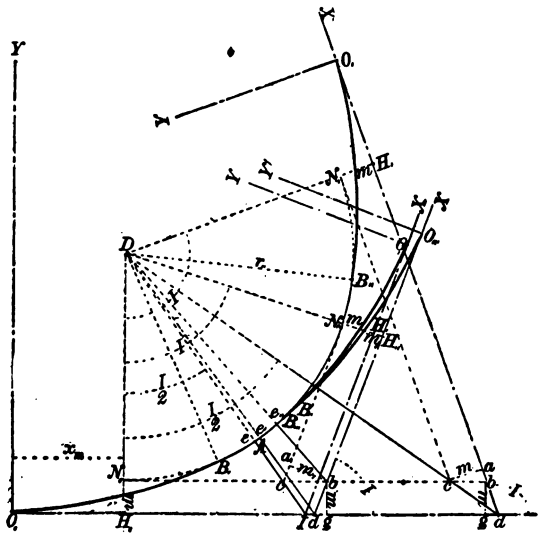


Fig. 5.

CASE 1.

When $m_{,} > m$ (by Fig. 5), if $I < 90°$,

$$O_{,,,}d_{,} = O_{,,,}H_{,,} + N_{,,}c_{,} - a_{,}c_{,} + b_{,}d_{,}$$
$$= x_{m_{,}} + T - m_{,} \cot I + m \operatorname{cose} I. \qquad (1)$$

$O_{,}d \quad m$

or in general calling,

$\mathbf{O_{,,,}d_{,}}$ or $\mathbf{O_{,}d = t_{,}}$; $\mathbf{t_{,} = x_{m_{,}} + T \mp m_{,}}$

the + sign being used when $I > 90°$:

$I < 90°$.

If $I < 90°$, $O_{,}d_{,} = O_{,}H_{,} + N_{,}c_{,}$

$x_m + T + m_{,}$ cose $I - m$

If $I > 90°$, $O_{,}d = O_{,}H_{,} + N_{,}c$ -

$x_m + T + m_{,}$ cose $I + n$

In general calling,

$\mathbf{O_{,}d_{,}}$ or $\mathbf{O_{,}d = t}$; $\mathbf{t = X_m + T + m_{,}}$

using + when $I > 90°$ and − wher

CASE 2.

If $\mathbf{m = o}$ in equation 4, we have

$\mathbf{t_{,} = T + m_{,}}$ **cose I,** $\mathbf{t = x_m +}$

CASE 3.

If $\mathbf{m = m}$ in equation 4,

$t_{,} = x_{m_{,}} + T + m_{,}$ (cos

$T + m$ (cose

but if $I > 90°$,

$$\cos I = -\cos I \text{ and } \cot I = -\cot I$$

$$\operatorname{cose} I + (-\cot I) = \frac{1 - \cos I}{\sin I},$$

which we have seen $= \tan \frac{1}{2}I$. **Hence when $\mathbf{m_{,} = m}$,**

$$\mathbf{t_{,} = t = x_m + T + m \tan \tfrac{1}{2}I = x_m + (r_{,,} + m) \tan \tfrac{1}{2}I} \quad (11)^*$$

(See App. I)

is a general equation whether I be greater or less than 90°. If in (11) we make x_m and m each $= o$, then $t = T = r_{,,} \tan \frac{1}{2}I =$ the semi-tangent for a simple circular curve.

EXAMPLE 10. Given $L = 210$; $r = 818.8$; $m_{,} = 2.23$; $I = 40°$; $O_{,}H_{,} = 104.93$; to find t (Fig. 5):

$t = x_{m_{,}} + (r_{,,} + m_{,}) \tan \frac{1}{2}I = 104.93 + 821.03 \times .36397 = 403.75$

With the same data

§ 12. TO FIND THE EXTERNAL SECANT.

CASE 1.

When $\mathbf{m_{,} > m}$,

$$\frac{H_{,}d_{,}}{DH_{,}} = \frac{T + m_{,} \operatorname{cose} I \pm m \cot I}{r_{,,} + m} = \tan H_{,}Dd_{,};$$

$$\mathbf{e_{,}d_{,} = (r_{,,} + m) \sec H_{,}Dd_{,} - r_{,,}.} \quad (12)$$

---

* If we wish to unite two grades by a vertical curve, then, if in Eq. 44, we make $\frac{1}{r_{,}} = o$

and

$$\sin \phi = \sin \tfrac{1}{2}I$$
$$L = 2r_{,,} \sin \tfrac{1}{2}I$$
$$x = L\,(1 -)$$
$$y = \frac{L^2}{6r_{,,}}$$

and Eq. 11, Prob. 1 $\quad t = x_m + (r_{,,} + m) \tan \frac{1}{2}I$

in which $\frac{1}{2}I = \frac{1}{2}$ the angle at which the grades intersect, $r_{,,}$ should have a value of from 5730 to 11460, and represents the minimum radius of curvature at the middle point of the vertical curve, x and y ordinates from the origin or terminus of the vertical curve to its middle point or to any other point.

CASE 2.

O,

$$\frac{N_{\prime}b_{\prime}}{r_{\prime\prime}} = \frac{T + m_{\prime} \operatorname{cose} I}{r_{\prime\prime}} = \tan N_{\prime}Db_{\prime} ;$$

$$e_{\prime\prime}b_{\prime} = r_{\prime\prime} \sec N_{\prime}Db_{\prime} - r_{\prime\prime}. \qquad (13$$

CASE 3.

$$= m, \; e_{\prime}d_{\prime} = (r_{\prime\prime} + m) \sec \tfrac{1}{2}I - r_{\prime\prime} \qquad (14$$

q. 14, we make $m = o$, we have for the externa
a simple circular curve:

$$ed = r_{\prime\prime} (\sec \tfrac{1}{2}I - 1) = ec.$$

ish the transition curves $B_{\prime\prime\prime}O_{\prime\prime\prime}$ and $B_{\prime}O_{\prime\prime}$ t
tangent to each other at some point A with
radius of curvature $= r_{\prime\prime} =$ the radius of th
ortion $B_{\prime\prime\prime}B_{\prime}$, with $B_{\prime\prime\prime}B_{\prime}$ elided, then $B_{\prime\prime\prime}$
with $B_{\prime}$.

$$= \alpha = I - \frac{L_{\prime}}{2r_{\prime\prime}}, \; B_{\prime\prime\prime}DN_{\prime\prime} = \alpha_{\prime} = L - \frac{L}{2r_{\prime\prime}}, \; (15$$

$$\alpha_{\prime} + \alpha = I \qquad (16$$

radius = 1 before entering the formulas, and after calculations are made the result expressed again in degrees and minutes to apply formulas for semi-tangents, external secants, etc.

PROBLEM II.

TO FIND THE LOCATION OF THE OFFSET "f".

§ 13. Given two curves with radii $r_{,}$ and $r_{,,}$, a distance $D_{,}D_{,,} = d$ joining the points $D_{,}$ and $D_{,,}$ also the angles $BD_{,}D_{,,} = \rho$ and $CD_{,,}D_{,} = \theta$, to find the points $A_{,}$ and $A_{,,}$ at which a line drawn through the centers $r_{,}$ and $r_{,,}$ at B and C will cut the curves $A_{,}D_{,}$ and $A_{,,}D_{,,}$.

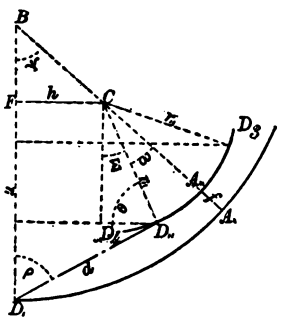


Fig. 6.

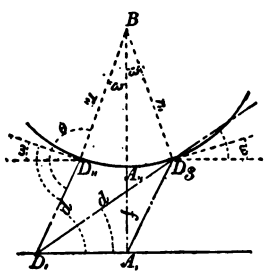


Fig. 7.

From Fig. 6 we have

$$FC = h = d \sin \rho - r_{,,} \sin (180 - (\rho + \theta)), \qquad (1)$$

$$BF = r_{,} - [d \cos \rho + r_{,,} \cos (180 - (\rho + \theta))]$$

$$\frac{FC}{BF} = \tan \Psi = \frac{d \sin \rho - r_{,,} \sin (180 - (\rho + \theta))}{r_{,} - [d \cos \rho + r_{,,} \cos (180 - (\rho + \theta))]} \qquad (2)$$

and since $\qquad BC = r_{,} - (r_{,,} + f),$

hich ω is the arc of a circle with radius

$\Sigma > \Psi$, then the point $A_{,,}$ lies betwe
the distance $A_{,,}D_3$ is measured fro
to $A_{,,}$.

$\Psi > \Sigma$, the point $A_{,,}$ lies beyond $D_{,}$
e $A_{,,}D_{,,}$ is measured from $D_{,,}$ to $A_{,,}$
blished. On a perpendicular to a tan
$_{,}$ and establish $A_{,}$.

hen f is small, the direction of the
estimated near enough. The metho
$B_{,,}$, in Fig. 2, has already been indicat
$_{,}$ is tangent. Fig. 7 applies.

EXAMPLE 11. $r_{,,} = 1000$; $r_{,,} = 600$; $d = 30$
$f = 50_{,}$:

irst, to find $h = d \sin \rho - r_{,,} \sin (180 - ($
$69 - 60 \times .24192 = 136.75 = FC$.

Second, to find $\sin \Psi = \dfrac{h}{r_{,} - (r_{,,} + f)} = \dfrac{}{1000}$

$$= \frac{136.75}{350} = 3097; \frac{FC}{BC} =$$

PROBLEM III.

COMPOUND CURVES.

§ 14. Given two circular curves with radii $r_{\prime}$ and r_3, respectively, whose centers are apart a distance $AC = b = r_1 - (r_3 + f_{\prime\prime})$, and which are separated from each other a distance $D_{\prime}D_{\prime\prime} = f_{\prime\prime\prime}$. It is desired to introduce between them a third curve with radius $r_{\prime\prime\prime}$, less than $r_{\prime}$ and greater than r_3 and to join the curve with radius $r_{\prime\prime}$ with those having radii $r_{\prime}$ and r_3 by means of transition curves:

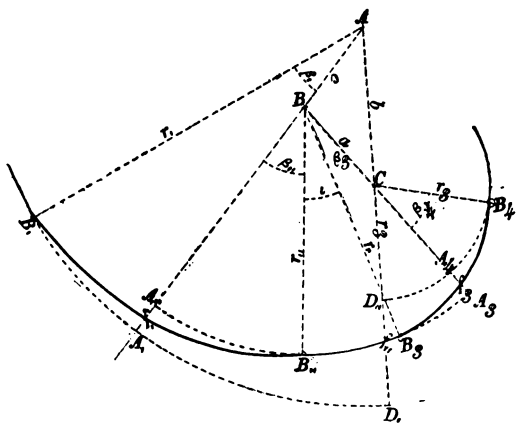


Fig. 8.

By the figure $AD_{\prime} = AA_{\prime} = AB_{\prime} = r_{\prime} = AC + CD_{\prime\prime} + D_{\prime\prime}D_{\prime} = b + r_3 + f_{\prime\prime\prime}$; whence

$$AC = b = r_{\prime} - (r_3 + f_{\prime\prime}); \tag{1}$$

$$AB = c = r_{\prime} - (r_{\prime\prime} + f_{\prime}); \tag{2}$$

$$BC = a = r_{\prime\prime} - (r_3 + f_3); \tag{3}$$

a, b and c form the sides of a triangle ABC in which

$$f_{\prime} = \frac{L_{\prime}^2}{24}\left(\frac{1}{r_{\prime\prime}} - \frac{1}{r_{\prime}}\right); \; f_3 = \frac{L_3^2}{24}\left(\frac{1}{r_3} - \frac{1}{r_{\prime\prime}}\right); \tag{4}$$

$$\text{Sin } B = \frac{b}{a} \sin A;$$

$$\text{Sin } C = \frac{c}{a} \sin A.$$

ducing each of the above angles to an arc as in
rt of this book, we have

$$A_{\prime}D_{\prime} = r_{\prime}A; \; A_4D_{\prime\prime} = r_3C; \; A_{\prime\prime}A_3 = r_{\prime\prime} \; ($$

e arc

$$\beta_4 = \frac{L_3}{2r_3} = \frac{P}{2r_3}\left(\frac{1}{r_3} - \frac{1}{r_{\prime\prime}}\right);$$

$$\beta_3 = \frac{L_3}{2r_{\prime\prime}} = \frac{P}{2r_{\prime\prime}}\left(\frac{1}{r_3} - \frac{1}{r_{\prime\prime}}\right);$$

the same way,

$$\beta_{\prime\prime} = \frac{L_{\prime}}{2r_{\prime\prime}} = \frac{P}{2r_{\prime\prime}}\left(\frac{1}{r_{\prime\prime}} - \frac{1}{r_{\prime}}\right);$$

$$* \; \beta_{\prime} = \frac{L_{\prime}}{2r_{\prime}} = \frac{P}{2r_{\prime}}\left(\frac{1}{r_{\prime\prime}} - \frac{1}{r_{\prime}}\right);$$

$$A_{\prime}B_{\prime} = r_{\prime}\beta_{\prime}; \; A_{\prime\prime}B_{\prime\prime} = r_{\prime\prime}\beta_{\prime\prime}; \; A_3B_3 =$$

$$A_4B_4 = r_3\beta_4;$$

$$B_{\prime\prime}B_3 = r_{\prime\prime}\,\iota; \; \dagger \; \iota = (180 - B) - (\beta_{\prime\prime}$$

$$B_{\prime}D_{\prime} = r_{\prime}A + r_{\prime}\beta_{\prime} = r_{\prime}\,(A + \beta_{\prime});$$

$$D_{\prime\prime}B_4 = r_3\,(C + \beta_4).$$

arc $C = .26054$; $A_4D_{\prime} = 2865 \times .06516 = 186.68$; $A_4D_{\prime\prime} = 716.3 \times .26054 = 186.77$; $A_{\prime\prime}A_3 = 1146 \times .3287 = 376.69$; $\beta_4 = \frac{90}{2(716.3)} = .6282$; $\beta_3 = \beta_2 = \frac{90}{2(1146)} = .03926$; $\beta_{\prime} = \frac{90}{2(2865)} = .0157$; $A_{\prime}B_{\prime} = A_{\prime\prime}B_{\prime\prime} = A_3B_3 = A_4B_4 = 45.0$; $\iota = 18° 48' - (2° 15' + 2° 15') = 14° 18'$ arc $\iota = .25017$; $B_{\prime\prime}B_3 = 1146 \times .25017 = 286.7$; $B_{\prime}D_{\prime} = 2865\ (.06516 + .0157) = 231.67$; $B_4D_{\prime\prime} = 716.3\ (.26054 + .06282) = 231.62$.

If we make $f_{\prime} = 0$ and $r_{\prime} = r_{\prime\prime}$, then $c = 0$, $C = 0$, $f_{\prime\prime}$ and $\beta_{\prime\prime} = 0$, $b = a$ and is coincident with it, $A = 180 - B$ and the problem reduces to uniting $A_{\prime\prime}A_3$ with $D_{\prime\prime}B_4$ by means of the transition curve. If $r_{\prime\prime} = r_{\prime}$, $c = 0$, $C = 0$, in which case $f_{\prime}$ and $f_3 = 0$; a is equal to and coincides with b and $A = 180 - B = 0$ equations for vers and sines $= 0$. The problem reduces to uniting curves $B_{\prime}D_{\prime}$ and $B_4D_{\prime\prime}$ by means of a transition curve whose length $L_2 = \sqrt{\frac{24 f_{\prime\prime}}{\frac{1}{r_{\prime}} - \frac{1}{r_3}}}$

If f_3 and $f_{\prime} = 0$, while $r_{\prime}$, $r_{\prime\prime}$ and r_3 retain their values, the transition curves disappear and the curve with radius $r_{\prime\prime}$ compounds at A_4 and $A_{\prime}$ with the curves having radii $r_{\prime}$ and r_3.

$$180 - B \text{ is the central angle of } A_{\prime\prime}A_3; \tag{18}$$

$$C = \text{that of } D_{\prime\prime}A_4; \qquad A = \text{that of } A_{\prime}D_{\prime}. \tag{19}$$

If the compound circular curve, Prob. III, be of two centers with radii $r_{\prime\prime}$ and $r_{\prime}$, then Eq. 12 and 13 give the value of $\beta_{\prime}$ and $\beta_{\prime\prime}$ if the central angles be not exceeding 10 or 15 degrees and the radii do not exceed 955 feet; otherwise

Let $\alpha_{\prime\prime} - \alpha_{\prime} = \beta_{\prime\prime} + \beta_{\prime} = I$ and $r_{\prime\prime}$ and $r_{\prime}$ be known; then, Eq. (6) page 5

$$P = \frac{2(\alpha_{\prime\prime} - \alpha_{\prime})}{\frac{1}{r_{\prime\prime}^2} - \frac{1}{r_{\prime}^2}} \tag{20}$$

and by (6)

$$\alpha_{\prime\prime} = \frac{P}{2r_{\prime\prime}^2},$$

$$\alpha_{\prime} = \frac{P}{2r_{\prime}^2},$$

and f computed by Eq. (37) then by § 4

$\beta_{\prime\prime}$ and $\beta_{\prime}$ can be obtained, whence $r_{\prime\prime}\beta_{\prime\prime}$ = the length of one branch of the curve and $r_{\prime}\beta_{\prime}$ the other. Then to find the *B. C.*, *C. C.* and *E. C.* of the compound circular curve, having

$$\alpha_{\prime\prime} - \alpha_{\prime} = \beta_{\prime\prime} + \beta_{\prime} = I. \tag{21}$$

By a similar process we may find $b_{//}$ when
ni-tangent adjacent $\beta_{//}$ and $T_{//} + b_{/} =$ the sem
which fixes the $B.\ C.$ and $E.\ C.$ and $r_{/}\beta_{/}$ tl
rve to $C.\ C.$ The distance $r_{/}\beta_{/}$ assumed to be
rve.*

If it be desired to introduce transition curve
this compound curve, the formulas of Proble
writing

$$T_{//} + b_{//} \text{ and } T + b_{/} \text{ for } T.$$

en the rate of curvature of the branches of
er from each other by less than two degrees
the transition curves may be omitted at the
l introduced only at the ends where the curv
gents.
The semi-tangent to a compound curve of m
may be computed by latitudes and departure

Case 2.

f it be desired to introduce a third circular
joining the other two fixed circular curves, b
, all turning in the same direction, then tl
cated in Fig. (8a) and the solution similar to
fixed value from which $f_{/}$, f_3, $L_{/} = B_{/}B_{//}$ an
puted. If, however, we wish to displace tl
e by spiral arcs

length of the spiral arcs computed from this data. Since $\beta_{,,} : \beta_3 = r_3 : r_{,}$ then $\frac{L_{0,}}{2} = r_{,,}\beta_{,,} = A_{,,}O_{,}$, $\frac{L_{03}}{2} = r_{,,}\beta_3 = A_3O_{,,}$.

$A_3O_{,,} + A_{,,}O_{,} < A_{,,}A_3$, generally, by an amount $O_{,}O_{,,}$. Make $O_{,}O = \frac{O_{,}O_{,,}r_3}{r_{,} + r_3}$ and $O_{,,}O = O_{,}O_{,,} - O_{,}O$ then $2\left(\frac{L_{0,}}{2} + O_{,}O\right) = L_{,}$

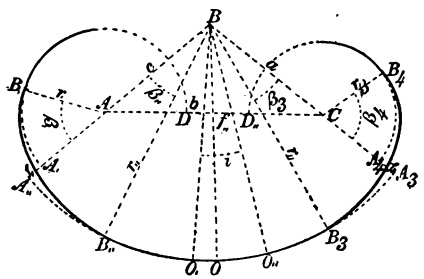


Fig. 8a.

$2\left(\frac{L_{03}}{2} + O_{,,}O_{,}\right) = L_3$ nearly, compute $f_{,}$ and f_3 and make the sides of the triangle, AB and AC, respectively $= r_{,,} - (r_{,} + f_{,})$ and $r_{,,} - (r_3 + f_3)$ and compute $OA_{,,}$ and OA_3 repeatedly till $= xf_{,}$ and xf_3, or $O_{,}O_{,,} = 0$ (or iota $= 0$). If we make $r_{,,}$ infinitely great, then $A_{,,}A_3$ will be a straight line and O the common origin of the spirals in which $r_{,} + r_3 \pm f_{,,} = xm_{,} + xm_3$ and we would have a special value of P for these particular curves. For a compound circular curve $f_{,}$ and $f_3 = 0$ as in the first part of this case.

PROBLEM IV.

§ 15. Given two curves turning in the same direction, whose centers are a distance apart $= D_{,}D_{,,}$ and whose radii are $r_{,}$ and $r_{,,}$. To fix the position of a tangent $A_{,}A_{,,,}$ and connect it with the circular curves $A_{,,}C_{,}$ and $A_4C_{,,}$ by means of transition curves having a fixed value of P (Fig. 9). Let $C_{,}C_{,,}$ be a line joining any two points $C_{,}$ and $C_{,,}$ of the circular curves with radii $r_{,}$

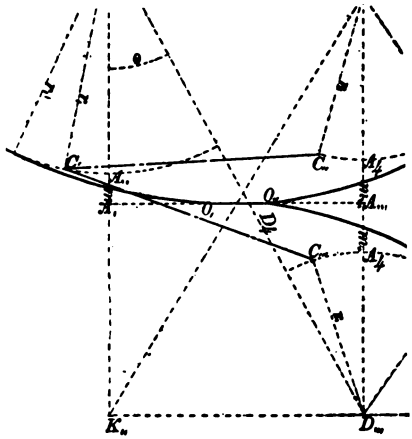


Fig. 9.

$r_{,,}$. If from $D_{,}$ and $D_{,,}$ we let fall perper
ginary tangent passing through the c
sition curves, by the conditions of the
th of these perpendiculars will be:

$C_iD_iA_{ii}$ = difference of $C_iD_iD_{ii}$ and θ. $C_iD_iA_{ii}$ reduced to arc and multiplied by r_i = the distance C_iA_{ii} to establish A_{ii}; from A_{ii} lay off m_i normal to the circle and establish A_i. In the same way $C_{ii}D_{ii}A_4$ = the difference between $180° - \theta$ and $C_{ii}D_{ii}D_i$. $C_{ii}D_{ii}A_4$ multiplied by r_{ii} = arc distance $C_{ii}A_4$ from C_{ii} to A_4 to establish A_4, from which lay off m_{ii} and establish A_{iii}; then a line through A_iA_{iii} is the required tangent.

The distance A_iO_i to the origin $O_i = x_i - r_i \sin \alpha_i$ and the distance $A_{iii}O_{ii} = x_{ii} - r_{ii} \sin \alpha_{ii}$; whence the distance

$$\mathbf{O_iO_{ii}} = \mathbf{D_iD_{ii}} \sin\theta - [(\mathbf{x}_{ii} - \mathbf{r}_{ii} \sin \alpha_{ii}) + (\mathbf{x}_i - \mathbf{r}_i \sin \alpha_i)]. \quad (3)$$

If we wish the distance O_iO_{ii} = zero, make

$$\mathbf{D_iD_{ii}} = [(\mathbf{x}_{ii} - \mathbf{r}_{ii} \sin \alpha_{ii}) + (\mathbf{x}_i - \mathbf{r}_i \sin \alpha_i)] \operatorname{cose} \theta. \quad (4)$$

This gives the shortest distance possible between the centers of the circular curves when transition curves are introduced. The transition curves may have different values of P provided $A_iO_i + A_{iii}O_{ii}$ = or < than A_iA_{iii}. **If the curves are reversed,** $D_iK_{ii} = (r_i + m_i) + (r_{ii} + m_{ii})$ and we find the value of θ by completing the traverse $C_iC_{iii}D_{iii}D_iC_i$.

EXAMPLE 13. Given $D_iC_iC_{ii}$ = 85°; $C_iC_{ii}D_i$ = 100; C_iC_{ii} = 300: r_i = 1146; r_{ii} = 573. First, to find $D_iD_{ii} = D_4$. By traverse with D_i as initial point.

From	Course	Distance	Lat. $N+$	Lat. $S-$	Dep $E+$	Dep $W-$	To
D_i	South	1146.0	.....	1146.0	.....	.....	C_i
C_i	$N85° E$	300.0	26.1	.....	298.9	.....	C_{ii}
C_{ii}	$N\ 5° E$	573.0	570.8	.....	49.9	.....	D_{ii}

$$+ 596.9 \quad - 1146.0 \quad + 348.8$$
$$596.9$$
$$- 549.1 \quad + 348.8$$

$\tan C_{\prime}D_{\prime}D_{\prime\prime}$ -

Dif. lat.

$32° 25' - 28° 25' = 4°$ $\therefore$ $D_{\prime}A_{\prime} = r_{\prime}$

$A_{\prime\prime}A_{\prime} = m_{\prime} = .13'$. By the table $C_{\prime\prime}$

$D_{\prime\prime}A_{\prime\prime\prime} = r_{\prime\prime} + m_{\prime\prime}$; parallel to $D_{\prime}A_{\prime}$; bears

$A_4A_{\prime\prime\prime} = 1.05$ $A_{\prime}A_{\prime\prime\prime} = D_{\prime\prime}K_{\prime}$

$= A_4A_{\prime\prime\prime} = 650.5 \times .4759$

$O_{\prime}O_{\prime\prime} = D_{\prime}D_{\prime} \sin \theta - [(x_{\prime\prime} - r_{\prime\prime} \sin \alpha$

Let $x_{\prime\prime} = 119.90$; $\alpha_{\prime\prime} = 6°$ $x_{\prime} = 6$

$O_{\prime}O_{\prime\prime} = 309.5 - [59.95 + 30] = 309.5$

to make $O_{\prime}O_{\prime\prime} = 0$

$$\tan \theta = \frac{89.95}{572.08} = .15273; =$$

$A_{\prime}A_{\prime\prime\prime} = 89.95 \cos \theta$, or $D_{\prime}D_{\prime\prime\prime} = 89.9$

PROBLEM V

OLD TRACK.

§ 16. To introduce the transi ment where circular curves have

In Fig. 10, suppose ABC, a sir have been run tangent to the line $r_{\prime} = D_{\prime}A = D_{\prime}B$; it is desired to i ...ture has a radius

AB = the distance to measure from $A = PC$ to locate B; $BA_{\prime\prime}$ the distance to measure from B on $BA_{\prime\prime}$ to locate $A_{\prime\prime}$. From m and $r_{\prime\prime}$ we have

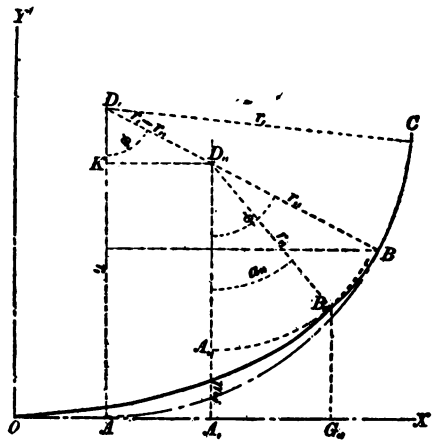


Fig. 10.

$$2r_{\prime\prime}a_{\prime\prime} = L = \sqrt{24mr_{\prime\prime}}, \text{ squaring, } 4r_{\prime\prime}^2a_{\prime\prime}^2 = 24mr_{\prime\prime}$$

or
$$a_{\prime\prime}^2 = \frac{6m}{r_{\prime\prime}}; \; a_{\prime\prime} = \sqrt{\frac{6m}{r_{\prime\prime}}}; \; r_{\prime\prime}a_{\prime\prime} = A_{\prime\prime}B_{\prime\prime} = \frac{L}{2}$$

$$\mathbf{r}_{\prime\prime}(\phi_{\prime\prime} - a_{\prime\prime}) = \mathbf{BB}_{\prime\prime}; \; (\mathbf{r}_{\prime} - \mathbf{r}_{\prime\prime}) \sin\phi = \mathbf{A}_{\prime}\mathbf{A}.$$

The rate of curvature of $r_{\prime\prime}$ should not be more than from 1° to 2° greater than that of $r_{\prime}$ (when possible) for curves of a curvature less than 10°; 2° to 3° difference for 10° to 15° rate of curvature; 3° to 5° difference for 15° to 20° rate of curvature.

EXAMPLE 14. Given $r_{\prime} = 1146$, $r_{\prime\prime} = 955$, $m = 1.52$, to find ϕ; $a_{\prime\prime}$, AB, $BB_{\prime\prime}$ and L

$$\text{ver}\,\phi = \frac{m}{r_{\prime} - r_{\prime\prime}} = \frac{142}{1146 - 955} = 0.00743 = \text{ver } 6° \, 59_{\prime}; \; \text{arc } \phi = .12188$$

EXPLANATION OF THE

BY RECTANGULAR CO-ORDI

§ 17. Tables 1 to 7 give values f
ransition curve by the method of r
nates. They are equally applicable fo
with a circular curve, or curves hav
by means of the transition curve. *I*
taken separately from the same colun
and $a_{,,}$ and their difference will be th
for the length and central angle resp
eral ordinates, x, y, $x_{,}$, $y_{,}$, are laid o
origin (Fig. 2) with arc $B_{,}A_{,}$ or B
same as if $B_{,}$ or $B_{,,}$ were written for
successive stations were B, C, D, e
etc., successive points x, $x_{,,}$, $x_{,,,}$, etc
y, $y_{,,}$, $y_{,,,}$, etc., values normal to t
$A_{,,}B_{,,}$ in the same manner as if
P and v are taken of such values a
fractions in L. L is supposed to
the chords are

The method of laying out these co-ordinates is shown in Fig. 4, in which the origin A corresponds to the point E in this example, and $AD_{\prime}$ becomes a curved axis with a radius of 1432.5.

If the transition curve were laid off from $B_{\prime\prime}A_{\prime\prime}$, Fig. 2 as axis and $B_{\prime\prime}$ as origin, the above values x, etc., and y, etc., would be just the same except they would be laid off from the convex side of $A_{\prime\prime}B_{\prime\prime}$ instead of from the concave side, as was the case with $A_{\prime}B_{\prime}$ as axis. If the curvature of the circular curve is of fractional degree the value of f and the last values of x, y and a will have to be computed by the formulæ at the head of the respective columns in Spiral Table I.

BY DEFLECTION.

(Or Polar Co-ordinates.)

§ 18. Given a tangent at any point of a transition curve as D to locate any other points as A, B, C, E, F, G and H. As in the case of the Tables for rectangular co-ordinates they are equally applicable for locating the points of the transition curve uniting a tangent to the circular curve or circular curves of different radii with each other. (Fig. 3.)

EXAMPLE 2. Let $L_{\prime\prime} - L_{\prime} = L = 120$ be the length of the transition curve; $r_{\prime} = 1910$; $r_{\prime\prime} = 716.3$ the radii of the circular curves to be united by L by deflections from a tangent at D, where the curvature corresponds to $r_{\prime}$. The tangent will be common to the circle and transition curve whose rate of curvature "D" (Table II) = 3°. DE, EF, FG, etc., being chords of the transition curve each = 30 feet. The tangent at D is a tangent to the circular arc with $r_{\prime} = 1910$. Then by the formula $\omega = L[D\Delta + N \times 0°00.1']$ in which, if we write $L = 30$, 60, 90 and 120; and $N = 1$, 2, 3 and 4 successively, $D = 3$, $\Delta = 0.3'$, then the deflection from tangent at

D to locate E is $\omega = 30\,[3 \times .3' + 1 \times 0°\,00.1'] = 0°\,30'$

D to locate F is $\omega = 60\,[3 \times .3' + 2 \times 0°\,00.1'] = 1°\,06'$

D to locate G is $\omega = 90\,[3 \times .3' + 3 \times 0°\,00.1'] = 1°\,48'$

D to locate H is $\omega = 120\,[3 \times .3' + 4 \times 0°\,00.1'] = 2°\,36'$

D to locate C is $\omega = 30\,[3 \times .3' - 1 \times 0^{\circ}$

D to locate B is $\omega = 60\,[3 \times .3' - 2 \times 0^{\circ}$

D to locate A is $\omega = 90\,[3 \times .3' - 3 \times 0^{\circ}$

If one of the chords be fractional and the cl
chord be fractional also.

Let $\frac{n}{F} = 15' = \frac{1}{2}$, $C = 1^{\circ}\,30' =$

n by (61)

$$\omega = L\left[D\Delta \pm \left(N + \frac{n}{F}\right) C \times 0^{\circ}\,0\right.$$

To locate any point, as $G + 15$ from tangent a

$$\omega = 105\,[3 \times .3 + (3 + \tfrac{1}{2}) \times 1\tfrac{1}{2} \times 0^{\circ}\,00.$$

To locate any point, as $A + 15'$ from tangent

$$\omega = 75\,[3 \times .3' - (2 + \tfrac{1}{2})\,1\tfrac{1}{2} \times 0^{\circ}\,00.1']$$

we have run the curve from A to D and chan
D in order to place the line of sight tangent
ht on A and deflect $0^{\circ}\,54'$ and we have a
:ilitate the use of the tables it is best to set t
d set the telescope on line AD, turn the ve
ıue deflection as tabulated, reading downwa
s points E, F, G, etc. If the curve is l
wards A then set the vernier at the angle ind
when backsight is on G from D deflect fron

TABLE OF CIRCULAR ARCS

Length of Circular Arcs at Radius = 1						Decimals of a Degree			
Sec.	Length	Min.	Length	Deg.	Length	Min.	Decimal	Sec.	Decimal
1	.000005	1	.000291	1	.017453	1	.01667	1	.00028
2	.000010	2	.000582	2	.034906	2	.03333	2	.00055
3	.000015	3	.000873	3	.052360	3	.05000	3	.00083
4	.000019	4	.001164	4	.069813	4	.06667	4	.00111
5	.000024	5	.001454	5	.087266	5	.08333	5	.00139
6	.000029	6	.001745	6	.104720	6	.10000	6	.00167
7	.000034	7	.002036	7	.122173	7	.11667	7	.00195
8	.000039	8	.002327	8	.139626	8	.13333	8	.00222
9	.000044	9	.002618	9	.157080	9	.15000	9	.00250
10	.000048	10	.002909	10	.174533	10	.16667	10	.00278
11	.000053	11	.003200	11	.191986	11	.18333	11	.00306
12	.000058	12	.003491	12	.209439	12	.20000	12	.00333
13	.000063	13	.003781	13	.226893	13	.21667	13	.00361
14	.000068	14	.004072	14	.244346	14	.23333	14	.00389
15	.000073	15	.004363	15	.261799	15	.25000	15	.00417
16	.000078	16	.004654	16	.279253	16	.26667	16	.00445
17	.000083	17	.004945	17	.296706	17	.28333	17	.00477
18	.000087	18	.005236	18	.314159	18	.30000	18	.00500
19	.000092	19	.005527	19	.331613	19	.31667	19	.00528
20	.000097	20	.005818	20	.349066	20	.33333	20	.00556
21	.000102	21	.006109	21	.366519	21	.35000	21	.00583
22	.000107	22	.006399	22	.383972	22	.36667	22	.00611
23	.000111	23	.006690	23	.401426	23	.38333	23	.00639
24	.000116	24	.006981	24	.418879	24	.40000	24	.00667
25	.000121	25	.007272	25	.436332	25	.41667	25	.00695
26	.000126	26	.007563	26	.453786	26	.43333	26	.00722
27	.000131	27	.007854	27	.471239	27	.45000	27	.00750
28	.000136	28	.008145	28	.488692	28	.46667	28	.00778
29	.000141	29	.008436	29	.506145	29	.48333	29	.00806
30	.000145	30	.008727	30	.523599	30	.50000	30	.00833
31	.000150	31	.009017	31	.541052	31	.51667	31	.00861
32	.000155	32	.009308	32	.558505	32	.53333	32	.00888
33	.000160	33	.009599	33	.575959	33	.55000	33	.00916
34	.000165	34	.009890	34	.593412	34	.56667	34	.00944
35	.000170	35	.010181	35	.610865	35	.58333	35	.00972
36	.000175	36	.010472	36	.628318	36	.60000	36	.01000
37	.000179	37	.010763	37	.645772	37	.61667	37	.01028
38	.000184	38	.011054	38	.663225	38	.63333	38	.01055
39	.000189	39	.011345	39	.680678	39	.65000	39	.01083
40	.000194	40	.011635	40	.698132	40	.66667	40	.01111
41	.000199	41	.011926	41	.715585	41	.68333	41	.01139
42	.000204	42	.012217	42	.733038	42	.70000	42	.01166
43	.000209	43	.012508	43	.750492	43	.71667	43	.01194
44	.000213	44	.012800	44	.767945	44	.73333	44	.01222
45	.000218	45	.013090	45	.785398	45	.75000	45	.01250
46	.000223	46	.013381	46	.802851	46	.76667	46	.01278
47	.000228	47	.013672	47	.820305	47	.78333	47	.01306
48	.000233	48	.013963	48	.837758	48	.80000	48	.01333
49	.000238	49	.014253	49	.855211	49	.81667	49	.01361
50	.000242	50	.014544	50	.872665	50	.83333	50	.01389
51	.000247	51	.014835	51	.890118	51	.85000	51	.01417
52	.000252	52	.015126	52	.907571	52	.86667	52	.01444
53	.000257	53	.015417	53	.925024	53	.88333	53	.01472
54	.000262	54	.015708	54	.942478	54	.90000	54	.01500
55	.000267	55	.016000	55	.959931	55	.91667	55	.01528
56	.000272	56	.016290	56	.977384	56	.93333	56	.01556
57	.000276	57	.016580	57	.994838	57	.95000	57	.01583
58	.000281	58	.016871	58	1.012291	58	.96667	58	.01611
59	.000286	59	.017162	59	1.029744	59	.98333	59	.01639
60	.000291	60	.017453	60	1.047198	60	1.00000	60	.0166

›, Subchord 50 Feet for One Degree Change of Curvature

BY RECTANGULAR CO-ORDINATES.

$\frac{L^2}{6r}$	$\frac{y}{4}$	$x - r \sin a$	$\frac{f}{2}$	$\sqrt{x^2 + y^2}$	$\frac{L}{2r} 57.3°$	$L - (x_f + \frac{1}{2}L)$		
y	f	x_f	y_f	$l.c.$	$a°$	d		
.....	.....	.....	.....	.....	.....	.....	.....	..
.01	.00	12.50	.00	25.00	0.07°	.....	.....	11
.07	.02	25.00	.01	50.00	0.25°	.....	.....	[illegible]
.24	.06	37.50	.03	75.00	0.57°	.....	.....	[illegible]
.58	.15	50.00	.07	100.00	1.00°	.....	.....	[illegible]
1.14	.28	62.50	.14	125.00	1.07°	.....	.....	[illegible]

BY DEFLECTIONS.

L	D	Transit on T'ng'nt	By Deflections toward A; $\omega = L\left[D\Delta - \left(N + \frac{n}{F}\right) C\, 0^\circ\, 00.1'\right]$									$\delta = \frac{2}{3}\alpha$	Gauge $= 4.7$ ft.	
0	0° A	At A	03′	10′	23′	40′	1°03′	1°30′	2°03′	2°40′	3°22½′	4°10′	0° A	250
25	30′	01½′	+25	06′	17½′	34′	55′	1°21½′	1°2½′	2°29′	3°10′	3°56′	30′	225
50	1° B	05′	05′	B	10′	25′	45′	1°10′	1°40′	2°15′	2°55′	3°40′	1° B	200
75	1°30′	11′	12½′	08½′	+25	14′	32½′	56½′	1°25′	1°59′	2°37½′	3°21′	1°30′	175
100	2° C	20′	22′	20′	12′	C	18′	40′	1°08′	1°40′	2°17½′	3°00′	2° C	150
125	2°30′	31′	35′	33½′	27½′	16′	+25	21′	47½′	1°19′	1°55′	2°36′	2°30′	125
150	3° D	45′	50′	50′	45′	35′	20′	D	25′	55′	1°30′	2°10	3° D	100
175	3°30′	1°01′	1°07½′	1°08½′	1°05′	56′	42½′	23½′	+25	29′	1°02½′	1°41′	3°30′	75
200	4° E	1°20′	1°27′	1°30′	1°27′	1°20′	1°16′	50′	27′	E	3°2½′	1°10′	4° E	50
225	4°30′	1°41′	1°50′	1°54′	1°52½′	1°46′	1°19′	1°35′	57½′	31′	+25	36′	4°30′	25
250	5° F	2°05′	2°15′	2°20′	2°20′	2°15′	2°05′	1°50′	1°30′	1°05′	35′	at F	5° F	0
Gauge $= 4.7$ ft.		$\delta = \frac{\alpha}{3}$	By Deflection from A; $\omega = L\left[D\Delta + \left(N + \frac{n}{F}\right) C\, 0^\circ\, 00.1'\right]$									Transit on T'ng'nt	D	L

TABLES.

hord = 30 Feet for One Degree Change of Curvatu

BY RECTANGULAR CO-ORDINATES.

	f	*xf*	*w*	*l.c.*	*a°*	*d*	
...	.....	.....	.....	.....	.....	.....	.....
3	.01	15.00	.00	30.00	0.15°	.....	.....
1	.05	30.00	.03	60.00°	0.60°	.....	.....
0	.10	37.50	.05	75.00	0.95°	.....	.....
1	.18	45.00	.09	90.00	1.35°		
2	.28	52.50	.14	105.00	1.85°		

BY DEFLECTIONS.

L	D	Transit on Tangent	By Deflections toward A; ω =									δ	Gauge = 4.7 ft.		
0	0° A	At A	06′	24′	37½′	54′	1°13½′	1°36′	2°01½′	2°30′	3°01½′	3°36′	0°	A	180
30	1° B	03′	B	15′	27½′	42′	1°00′	1°21′	1°45′	2°12′	2°42′	3°15′	1°	B	150
60	2° C	12′	12′	C	10½′	24′	40½′	1°00′	1°22′	1°48′	2°16½′	2°48′	2°	C	120
75	2°30′	19′	20′	10′	+15	13′	28½′	47′	1°09′	1°34′	2°01½′	2°32′	2°30′		105
90	3° D	27′	30′	21′	12′	D	15′	33′	54′	1°18′	1°45′	2°15′	3°	D	90
105	3°30′	37′	41′	34′	25½′	14′	+15	17′	37½′	1°01′	1°27′	1°56′	3°30′		75
120	4° E	48′	54′	48′	40½′	30′	16½′	E	19½′	42′	1°07½′	1°36′	4°	E	60
135	4°30′	1°01′	1°08′	1°04′	57′	47′	34½′	19′	+15	22′	46½′	1°14′	4°30′		45
150	5° F	1°15′	1°24′	1°21′	1°15′	1°06′	54′	39′	21′	F	24′	51′	5°	F	30
165	5°30′	1°31′	1°41′	1°40′	1°34½′	1°26′	1°15′	1°01′	43½′	23′	+15	26′	5°30′		15
180	6° G	1°48′	2°00′	2°00′	1°55½′	1°48′	1°37½′	1°24′	1°07½′	48′	25′	At G	6°	G	0
Gauge = 4.7 ft.		δ	By Deflections from A; ω =									Transit on T'ng'nt	D		L

ubchord = 20 Feet for One Degree Change of Curvatu

BY RECTANGULAR CO-ORDINATES.

y	f	x_f	y_f	$l.c.$	$a°$	d		
.....	.....	.....	.....	.....	.....	.....	.....	..
.01	.....	20.00	.001	20.00	0.10°	.....	.....	
.09	.02	20.00	.01	40.00	0.40°	.....	.....	
.31	.08	30.00	.04	.....	0.90°	.....	.....	
.74	.19	40.00	.10	80.00	1.60°	.....	.....	
1.45	.36	50.00	.18	100.00	2.50°	.....	.....	
2.51	.63	59.99	.32	119.98	3.60°	.01	.....	

BY DEFLECTIONS.

L	D		Transit on T'ng'nt	By Deflections toward A; ω =									δ	Gauge =4.7 ft.		
0	0°	A	At A	04′	16′	36′	1°04′	1°40′	2°24′	3°16′	4°16′	5°24′	6°40′	0°	A	200
20	1°	B	02′	B	10′	28′	54′	1°28′	2°10′	3°00′	3°58′	5°04′	6°18′	1°	B	180
40	2°	C	08′	08′	C	16′	40′	1°12′	1°52′	2°40′	3°36′	4°40′	5°52′	2°	C	160
60	3°	D	18′	20′	14′	D	22′	52′	1°30′	2°16′	3°10′	4°12′	5°22′	3°	D	140
80	4°	E	32′	36′	32′	20′	E	28′	1°04′	1°48′	2°40′	3°40′	4°48′	4°	E	120
100	5°	F	50′	56′	54′	44′	26′	F	34′	1°16′	2°06′	3°04′	4°10′	5°	F	100
120	6°	G	1°12′	1°20′	1°20′	1°12′	56′	32′	G	40′	1°28′	2°24′	3°28′	6°	G	80
140	7°	H	1°38′	1°48′	1°50′	1°44′	1°30′	1°08′	38′	H	46′	1°40′	2°42′	7°	H	60
160	8°	I	2°08′	2°20′	2°24′	2°20′	2°08′	1°48′	1°20′	44′	I	52′	1°52′	8°	I	40
180	9°	J	2°42′	2°56′	3°02′	3°00′	2°50′	2°32′	2°06′	1°32′	50′	J	58′	9°	J	20
200	10°	K	3°20′	3°36′	3°44′	3°44′	3°36′	3°20′	2°56′	2°24′	1°44′	56′	at K	10°	K	0
Gauge =4.7 ft.			δ	By Deflection from A; ω =									Transit on T'ng'nt	D		L

TABLES.

Subchord = 15 Feet for One Degree Change of Curvat

BY RECTANGULAR CO-ORDINATES.

y	f	x_f	y_f	$l.c.$	$a°$	d		
.....	.....	.....	.....	.....	.....	.....	.....	..
.05	.01	15.00	.00	30.00	.30°	.....	.....	1
.42	.11	30.00	.06	60.00	1.20°	.....	.....	1
.82	.20	37.50	.10	75.00	1.875°	.....	.....	1
1.41	.35	45.00	.18	90.00	2.70°	.....	.....	
2.24	.56	52.49	.28	104.99	3.675°	.01	.....	
3.35	.84	59.97	.42	119.97	4.80°	.03	.....	

BY DEFLECTIONS.

L	D		Transit on T'ng'nt	By Deflections toward A; ω =									δ	Gauge = 4.7 ft.		
0	0°	A	At A	12′	48′	1°15′	1°48′	2°27′	3°12′	4°03′	5°00′	6°03′	7°12′	0°	A	180
30	2	C	06′	C	30′	54′	1°24′	2°00′	2°42′	3°30′	4°24′	5°24′	6°30′	2°	C	150
60	4°	E	24′	24′	E	21′	48′	1°21′	2°00′	2°45′	3°36′	4°33′	5°36′	4°	E	120
75	5°	F	37½′	40½′	19½′	F	25½′	57′	1°34½′	2°18′	3°07½′	4°03′	5°04½′	5°	F	105
90	6°	G	54′	1°00′	42′	24′	G	30′	1°06′	1°48′	2°36′	3°30′	4°30′	6°	G	90
105	7°	H	1°13½′	1°22½′	1°07½′	51′	28½′	H	34½	1°15′	2°01½′	2°54′	3°52½′	7°	H	75
120	8°	I	1°36′	1°48′	1°36′	1°21′	1°00′	33′	I	39′	1°24′	2°15′	3°12′	8°	I	60
135	9°	J	2°01½′	2°16½′	2°07½′	1°54′	1°34½′	1°09′	37½′	J	43½′	1°33′	2°28½′	9°	J	45
150	10°	K	2°30′	2°48′	2°42′	2°30′	2°12′	1°48′	1°18′	42′	K	48′	1°42′	10°	K	30
165	11°	L	3°01½′	3°22½′	3°19½′	3°09′	2°52½′	2°30′	2°01½′	1°27′	46½′	L	52½′	11°	L	15
180	12°	M	3°36′	4°00′	4°00′	3°51′	3°36′	3°15′	2°48′	2°15′	1°36′	51′	at M	12°	M	0
Gauge = 4.7 ft.			δ	By Deflection from A; ω =									Transit on T'ng'nt	D		L

Subchord = 10 Feet for One Degree Change of Curvature

BY RECTANGULAR CO-ORDINATES.

y	f	x_f	y_f	$l.c.$	$\alpha°$	d		
.....	.....	.....	.....	.....	.....	.....	.....	..
02	.01	10.00	.00	20.00	0.20°	.....	.....	2
.19	.05	20.00	.03	40.00	0.80°	.....	.....	1
.63	.16	30.00	.08	60.00	1.80°	.....	.....	
1.49	.37	40.00	.19	79.99	3.20°	.....	.....	
2.90	.72	49.99	.36	99.97	5.00°	.....	.....	
5.02	1.25	59.97	.63	119.93	7.20°	.03	.....	
6.38	1.59	64.96	.80	129.90	8.45°	.04	.....	

BY DEFLECTIONS.

L	D		Transit on T'ng'nt	By Deflections toward A; ω =									δ	Gauge = 4.7 ft.		
0	0°	A	At A	08′	32′	1°12′	2°08′	3°20′	4°48′	5°38′	6°32′	7°30′	8°32′	0°	A	160
2	2°	C	04′	C	20′	56′	1°48′	2°56′	4°20′	5°08′	6°00′	6°56′	7°56′	2°	C	140
40	4°	E	16′	16′	E	32′	1°20′	2°24′	3°44′	4°30′	5°20′	6°14′	7°12′	4°	E	120
60	6°	G	36′	40′	28′	G	44′	1°44′	3°00′	3°44′	4°32′	5°24′	6°20′	6°	G	100
80	8°	I	1°04′	1°12′	1°04′	40′	I	56′	2°08′	2°50′	3°36′	4°26′	5°20′	8°	I	80
100	10°	K	1°40′	1°52′	1°48′	1°28′	52′	K	1°08′	1°48′	2°32′	3°20′	4°12′	10°	K	60
120	12°	M	2°24′	2°40′	2°40′	2°24′	1°52′	1°04′	M	38′	1°20′	2°06′	2°56′	12°	M	40
130	13°	N	2°49′	3°07′	3°09′	2°55′	2°25′	1°39′	37′	N	41′	1°26′	2°15′	13°	N	30
140	14°	O	3°16′	3°36′	3°40′	3°28′	3°00′	2°16′	1°16′	40′	O	44′	1°32′	14°	O	20
150	15°	P	3°45′	4°07′	4°13′	4°03′	3°37′	2°55′	1°57′	1°22′	43′	P	47′	15°	P	10
160	16°	Q	4°16′	4°40′	4°48′	4°40′	4°16′	3°36′	2°40′	2°06′	1°28′	46′	At Q	16°	Q	0
Gauge = 4.7 ft.			δ	By Deflections from A; ω =									Transit on T'ng'nt	D		L

TABLES.

hord = 10 Feet for Two Degrees Change of Curvat

BY RECTANGULAR CO-ORDINATES.

f	x_f	y_f	$l.\ c.$	$\alpha°$	d		
.....	.....	.....	.....	.....	.....	.....	.
.01	10.00	.00	20.00	0.40°	.....	.....	
.09	20.00	.05	40.00	1.60°	.....	.....	
.32	30.00	.16	59.99	3.60°	.....	.....	
.75	39.99	.37	79.98	6.40°	.01	.....	
1.06	44.97	.53	89.92	8.10°	.03	.....	
1.45	49.95	.72	99.90	10.00°	.05	.....	

BY DEFLECTIONS.

L	D		Transit on T'ng'nt	By Deflections toward A; ω =									δ	Gauge =4.7 ft.		
0	0°	A	At A	16′	1°04′	2°24′	4°16′	5°24′	6°40′	8°04′	9°36′	11°16′	13°04′	0°	A	140
20	4°	C	08′	C	40′	1°52′	3°36′	4°40′	5°52′	7°12′	8°40′	10°16′	12°00′	4°	C	120
40	8°	E	32′	32′	E	1°04′	2°40′	3°40′	4°48′	6°04′	7°28′	9°00′	10°40′	8°	E	100
60	12°	G	1°12′	1°20′	56′	G	1°28′	2°24′	3°28′	4°40′	6°00′	7°28′	9°04′	12°	G	80
80	16°	I	2°08′	2°24′	2°08′	1°20′	I	52′	1°52′	3°00′	4°16′	5°40′	7°12′	16°	I	60
90	18°	J	2°42′	3°02′	2°50′	2°06′	50′	J	58′	2°04′	3°18′	4°40′	6°10′	18°	J	50
100	20°	K	3°20′	3°44′	3°36′	2°56′	1°44′	56′	K	1°04′	2°16′	3°36′	5°04′	20°	K	40
110	22°	L	4°02′	4°20′	4°26′	3°50′	2°42′	1°56′	1°02′	L	1°10′	2°28′	3°54′	22°	L	30
120	24°	M	4°48′	5°20′	5°20′	4°48′	3°44′	3°00′	2°08′	1°08′	M	1°16′	2°40′	24°	M	20
130	26°		5°38′	6°14′	6°18′	5°50′	4°50′	4°08′	3°18′	2°20′	1°14′	N	1°22′	26°	N	10
140	28°	O	6°32′	7°12′	7°20′	6°56′	6°00′	5°20′	4°32′	3°36′	2°32′	1°20′	at O	28°	O	0
Gauge =4.7 ft.			δ	By Deflection from A; ω =									Transit on T'ng'nt	D		L

.003	.000	1.50	.00	3.00	0.15°	...	573.0
.02	.005	3.00	.00	6.00	0.60°	...	286.5
.07	.02	4.50	.01	9.00	1.35°	...	191.0
.11	.03	5.25	.01	10.50	1.83°	...	163.7
.17	.04	6.00	.02	12.00	1.40°	...	143.2
.24	.06	6.75	.03	13.50	3.03°	...	127.3
.33	.08	7.50	.04	15.00	3.75°	...	114.6
.44	.11	8.25	.05	16.50	4.53°	...	104.2
.56	.14	9.00	.07	17.00	5.40°	...	95.5
.66	.15	9.45	.07	18.89	5.95°	...	90.9
.76	.19	9.90	.09	19.79	6.53°	...	86.8
.86	.22	10.35	.11	20.68	7.15°	...	83.0
.98	.24	10.80	.12	21.58	7.77°	...	79.7
1.10	.25	11.25	.12	22.48	8.43°	...	76.4
1.24	.31	11.70	.15	23.37	9.13°	...	73.4
1.39	.35	12.15	.17	24.27	9.77°	...	70.7
1.55	.39	12.60	.19	25.16	10.58°	...	68.1
1.73	.43	13.04	.21	26.06	11.35°	.01	65.9
1.89	.48	13.49	.24	26.79	12.20°	.01	63.[illegible]
2.03	.51	13.78	.25	27.53	12.70°	.02	62.[illegible]
2.18	.54	14.08	.27	28.13	13.27°	.02	60.[illegible]
2.30	.58	14.37	.29	28.72	13.82°	.03	59.[illegible]
2.43	.62	14.66	.31	29.32	14.40°	.04	58.[illegible]
2.60	.65	14.96	.32	29.91	15.00°	.04	57.[illegible]
2.76	.69	15.25	.34	30.50	15.60°	.05	56.[illegible]
[illegible]	[illegible]	15.55	.37	31.10	16.22°	.05	55.[illegible]

APPENDIX I.

PROBLEM.

Given: The altitude of two circumpolar stars, nearly opposite each other, the local time of Meridian passage of either star, their difference in time of Meridian passage and the polar distance of each star,

TO FIND THE LATITUDE, THE LOCAL TIME AND THE MERIDIAN OF THE PLACE.

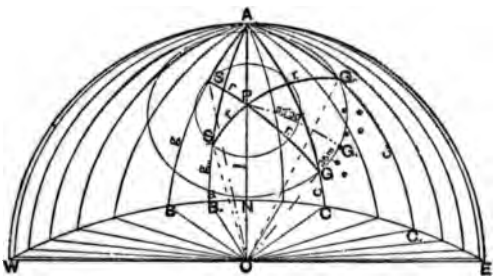


Fig. 11.

For convenience, let Polaris and Alioth be the stars observed; O the position of the observer and center of the celestial sphere. $ONEW$ be the north half of that portion of the celestial sphere above the horizon WNE, with A as zenith.

Let $GC = c$ or $G_{\prime\prime}C_{\prime\prime} = c_{\prime\prime}$ be the altitude of Alioth at the time of observation and $BS = g$ or $B_{\prime\prime}S_{\prime\prime} = g_{\prime\prime}$ the altitude of Polaris * 35 minutes later; then the plane passing through the points S, G and O will pass through the pole P and cut the celestial sphere in the line $r + r_{\prime}$ SG or $S_{\prime\prime}G_{\prime\prime}$, r † and $r_{\prime}$ being the polar distance respectively of Polaris and Alioth and $AP = s$ the co-latitude of the place O. Then the plane ASO or $AS_{\prime\prime}O$ will cut the side of the

---

* At this date.

† The position of these stars have a slight annual change with reference to the pole, which must be taken into account from year to year.

os $(90 - c) = \sin c$ and $\cos (90 - g) = \sin g$, l

$$\cos S = \frac{\sin c - \sin g \cos (r + r_t)}{\cos g \sin (r + r_t)} .$$

ıe angle S is common to the two triangles ASO

e now have r, S and $(90 - g)$ to determine A
ıe Latitude and P = the Hour Ang e (express

ıs $(90 - l) = \cos (90 - g) \cos r \pm \sin (90° - g)$

$$\sin l = \sin g \cos r \pm \cos g \sin r \cos S,$$

$-$ sign when $S > 90$; and since

$$\sin (90 - l) : \sin (90 - g) :: \sin S : \sin P$$

$$\cos l : \cos g :: \sin S : \sin P$$

$$\sin P = \frac{\cos g \sin S}{\cos l} .$$

represents the angle the plane OSP makes
at the time of observation on S and $P° \times 4$ t
ssed in minutes of sidereal time, from the plan
· triangle ASP we have

$$\sin A = \frac{\sin r \sin P}{\sin (90 - g)} = \frac{\sin r \sin P}{\cos g}$$

Example. Given $g = 40$; $c = 20$; $r = 33° 30'$ to find the latitude l. We first find the angle S by the formula,

$$\cos S = \frac{\sin c - \sin g \cos (r + r_{\prime})}{\cos g \sin (r + r)} = \frac{.3292 - .64279 \times .82181}{.76604 \times .56976}$$

$$= - \frac{.18623}{.43646} = - .42667 = \cos 115.15.$$

To find the latitude we have (a)

$$\sin l = \sin g \cos r \pm \cos g \sin r \cos S$$

using the $-$ sign (since $S > 90°$),

$\sin l = .64279 .9998 - .76604 \times .0215 \times 42667 = .642661 - 00702 = .63564 = \sin 39° 28'$, whence the Latitude $39° 28'$.

To find the local time we have (b)

$\sin P = \frac{\cos g \sin S}{\cos r} = \frac{.76604 \quad .90445}{.7720} = .89741 = \sin 63° 49'$, $P \times 4^m$

$= 63° 49' \times 4 = 4^h 15^m = 255$ sidereal minutes; $255 \times .99727 = 254$ mean solar minutes, or $P = 4^h 14^m$.

The time of observation was at 1 o'clock A.M., Dec. 1st, 1900, i.e. $13^h - 4^h 14^m = 8^h 46^m$ the time of meridian passage by the clock.

The local time of meridian passage was $8^h 40^m$ i.e. $8^h 46^m - 8^h 40^m = 0^h 06^m$ i.e. the clock was 6^m fast.

To find the Azimuth Angle $SAP = BON = A$, we have (c)

$\sin A = \frac{\sin r \sin P}{\cos g} = \frac{.0215 \times .7974}{.76604} = .02518$ whence $A = 1° 27'$

San Francisco, 1898.

ngitude 31 (2h 04m) West of Washington.

1905	Year 1906	Year 1907	Year 1908	Year 1909
M.	H. M.	H. M.	H. M.	H. M.
.4 P.M.	6 42.8 P.M.	6 44.3 P.M.	6 45.7 P.M.	6 43.2 P.M.
.0 "	4 40.4 "	4 41.3 "	4 43.3 "	4 40.8 "
.5 "	2 49.9 "	2 51.4 "	2 48.9 "	2 50.3 "
.5 "	0 47.9 "	0 49.4 "	0 46.8 "	0 48.3 "
.7 A.M.	10 46.1 A.M.	10 47.6 A.M.	10 45.1 A.M.	10 46.5 A.M.
.1 "	8 44.5 "	8 46.0 "	8 43.5 "	8 44.9 "
.7 "	6 47.1 "	6 48.6 "	6 46.1 "	6 47.5 "
.3 "	4 45.7 "	4 47.2 "	4 44.7 "	4 46.1 "
.9 "	2 44.3 "	2 45.8 "	2 43.3 "	2 44.7 "
.2 "	0 46.6 "	0 48.1 "	0 45.6 "	0 47.0 "

METHOD OF COMPUTING RADIUS.

The usual method of computing the radius of curvature of circular curves used in railroad location, is by the formula

$$r = \frac{\frac{1}{2}c}{\sin \frac{1}{2}D}, \tag{1}$$

in which c represents a chord of 50 feet in length, and D is the degree of the curve or central angle for a chord of 100 feet. By this method the radius does not vary inversely as the degree of the curve; while in the method of this book it is assumed to do so, and r is assumed to equal the radius of a one-degree curve divided by the degree of the proposed curve, or

$$r = \frac{5730}{D}. \tag{2}$$

These formulae give the same result for a five-degree curve for 50-foot chords, thus:

$$r = \frac{\frac{1}{2}c}{\sin \frac{1}{2}D} = \frac{25}{.021815} = 1146.00 \text{ and } r = \frac{5730}{5} = 1146.00,$$

from which we see, by the method of this book, the radius of a five-degree curve is the same computed by either formula. For less than 5° rate of curvature r eq. (1) $< r$ eq. (2) and for a rate of curvature greater than 5° r eq. (1) $> r$ eq. (2) for 50-foot stations.

Since the length of the chords is usually less than 50 feet, in the spiral the radius of curvature is assumed to vary inversely as the degree of the curve, and the same radius may be used for the main curve, by making its chords less than 50 feet, the proper length to be determined, by computation, and tabulated for field use. It is scarcely worth notice (= .002 per degree of curve less than 50 feet for each degree in rate of curvature per 100 feet of arc).

TO DETERMINE THE NUMERICAL CONSTANT IN FORMULA (6o):

Place $0°00.1' = k$, then we have $\delta' = LNCk$

or

$$k = \frac{\delta'}{LNC}. \tag{1}$$

We find δ' by dividing eq. (21) by eq. (26), whence,

$$\tan \delta = \frac{y}{x} = \frac{\frac{L^2}{6r}\left(1 - \frac{L^2}{56r^2}\right)}{L\left(1 - \frac{L^2}{40r^2}\right)} +, \text{ or } \tan \delta = \frac{\frac{L}{6r}\left(1 - \frac{L^2}{56r^2} +\right)}{1 - \frac{L^2}{40r^2} +}.$$

[illegible] with two solutions ξ_1 and ξ_2 such that $\xi_1 \xi_2 = \frac{\mu_0 \mu_1}{\mu_2}$ [illegible]

· δ and $2L$ for L, then calling the numerical co

$$k_{,} = \frac{a' - \delta' n}{2LNC}$$

ng in a contrary direction from the same poi
s supplement. $k_{,}$ should always come out gre
alue of $k_{,}$ will be found to increase and
with α. For $L = 200$ feet, or less, and $\alpha =$
make $k_{,} = k = 0°00.1'$.
l be seen by a trial example that the differe
hod and that of eq. (61) for a curve with L
oint at 200 feet) and $\alpha = 16°$, is $0°00'03''$.
lifference increases with the central angle α.
with change points 200 to 300 feet apart, is
best for preparing a set of tables, though r
n field computations as eqs. (60) or (61).
ssuming "change points" from 100 to 250 fe
ı the deflection tables of this book may be su
ıd term in eq. (61) and the spiral extended in
ollowing table shows the results of computa
rmulae.
alue of the constant C in each particular case i

· C, in which $D_{,}$ represents the degree, or rate

P = 114,600.

D	L	a	x	y	$\tan \frac{y}{x}$	δ	a − δ	k	k′
2	40	24′	40.00	.09	0.00250	08.5′	16.5′	.1000′	.1000′
4	80	1°36′	79.99	.74	0.00925	31.8′	1°04.2′	.1000′	.1000′
6	120	3°36′	119.95	2.51	0.02092	1°12′	2°24′	.1000′	.1000′
8	160	6°24′	159.80	5.95	0.03723	2°08′	4°16′	.1000′	.1000′
10	200	10°00′	199.40	11.61	0.05822	3°20′	6°40′	.1000′	.1000′
12	240	14°24′	238.5	20°01′	0.08390	4°47.75′	9°36.25′	.09996′	.10003′
14	280	19°36′	276.7	31.66	0.11442	6°31.67′	13°04.33′	.09992′	.10006′
16	320	25°36′	313.7	46.99	0.14979	8°31′	17°05′	.09990′	.10010′
18	360	32°24′	348.7	66.32	0.19073	10°48′	21°36′	.09971′	.10012′
20	400	40°00′	380.9	89.89	0.23599	13°16.67′	26°43.33′	.09958′	.10021′
22	440	48°24′	409.6	117.72	0.28735	16°02′	32°22′	.09921′	.10047′

For the same value of δ, L and D, are inverse functions of each other (nearly), and k and k, remain the same and vary only with the deflection angles δ′ and a′ − δ′.

angle due to the circular portion of the cur
sd radius $r_{\prime\prime}$ and

$$B_{\prime}DB_{\prime\prime\prime} = I - (\alpha_{\prime} + \alpha),$$

, by making the second term in eq. (61) = 0, w

Δ = the deflection from a tangent at any
curve, as $B_{\prime}$ or $B_{\prime\prime\prime}$ to locate any other point o
etween $B_{\prime}$ and $B_{\prime\prime\prime}$ inclusive of the last point
may be.

SUPPLEMENT TO PROBLEM III.

the arc of that portion of the curve, between
lar and laid out by the method of circular curv
m tangent being computed by eq. (61), in whi
brackets, is made = 0, whence, it becomes ω =
be reduced from arc to degrees and minutes
al deflection.

THE PROCESS OF FIELD WORK.

inning at $O_{\prime}$, as instrument point, deflect from
cessive stations by formula (60) including $B_{\prime}$
the instrument at $B_{\prime}$, with backsight on $O_{\prime}$, de

Having located O_{iii}, set up instrument at O_{iii} and with back-sight on B_{iii} deflect

$$\delta' = LNC \times 0^\circ\, 00.1',$$

and the line of sight should coincide in direction and position with the tangent $O_{iii}X$, or the semi-tangent $O_{iii}d_i$.

Some prefer to run the spirals first, i.e., from O_i to B_i and from O_{iii} to B_{iii}, and the circular portion B_iB_{iii} last, thus throwing the closing error, if any, at B_i or B_{iii}.

Undoubtedly this makes a better adjustment when the points O_i and O_{iii} have been fixed previously by semi-tangents.

All change points should be established by *double centers* to eliminate errors of adjustment in the transit instrument.

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